ON THE INFLUENCE OF THE WIDTH OF THE TENSILE SPECIMEN.

Thesis presented to the University of Edinburgh

by

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for

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in the Department of Engineering.

PREFACE.

This Thesis is a record of original research carried out in the Engineering Laboratory of the University of Edinburgh mainly under the direction of Prof. Hudson Beare.

Some of the results appearing have already been communicated to the Royal Society of Edinburgh and the British Association for the Advancement of Science. These contributions, which were well received on presentation, have been printed verbatim, abstracted, or reviewed in several British and foreign technical Journals; and reference to the more useful data and conclusions is made in certain modern text-books on Materials. The greater part of the Thesis is, however, the outcome of more recent effort which included much confirmatory, but unrecorded, work.

The writer desires to express his thanks to Prof. Hudson Beare, to Messrs. David Colville & Sons, Ltd., of Motherwell, and to the Carnegie Trust for the Universities of Scotland.

EDINBURGH, October, 1924.

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ON THE INFLUENCE OF THE WIDTH OF THE TENSILE SPECIMEN.

1. Introduction.

It is well known that the strength and the ductility of a metal, as determined in the tensile test, depend not only upon the physical properties of the metal but also upon the form and dimensions of the test-bar. For example, abrupt variations of section, resulting in virtual shortening of the specimen, raise the apparent strength and lower the apparent ductility of the metal under test.

Again, owing to the phenomenon of contraction at rupture in ductile metals, the apparent ductility diminishes with increase of gauge-length. The extensions of two test-bars of the same linear sectional dimensions are therefore not comparable unless measured on the same gauge-length.

The extensions of bars, geometrically similar but differing in dimension, are comparable only by observing Barba's Law of Similarity - <u>Geometrically similar bodies of the same material, broken in identical conditions by tension, undergo similar deformations.</u> In practice it is exceedingly inconvenient to prepare all test-bars so that they shall be geometrically similar; but the recommendations of the British Engineering Standards Association go far towards effecting a successful compromise and establishing a much more rational procedure than had prevailed in the past.

Finally, there is the difficulty of obtaining comparable results from bars of the same sectional form but geometrically dissimilar and from bars of different sectional form. Within limits, this difficulty may be overcome by using for each bar a gauge-length proportional to the square root of its cross-sectional area.

Hence, in general, it would appear that it is the specimen of the metal, and not the metal itself, which is tested. The matter is one of great practical importance, for on the form and proportions of the test-piece may depend the acceptance or rejection of a large quantity of material.

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2. Object of Experiments.

The research originated in an attempt to re-establish the critical ratio of width to thickness which Barba has shown to give a maximum of elongation; but later, as the scope of the investigation widened, the object of the enquiry became the determination of the nature of the influence of the width of the specimen upon the apparent strength and ductility of the metal under test. To that end, material of constant thickness was employed in each experiment. The enquiry is thus confined to the case of rectangular test-bars; but, as the majority of tests made in steel works are carried out on flats, the question is one of more than purely scientific interest.

The research, so far as this Thesis is concerned, comprises three distinct investigations. In the first place, a preliminary enquiry was made into the effects of varying the width of specimens cut from $\frac{1}{4}$-inch mild steel plate. This material was obtained locally and, though of unknown origin, proved to be of uniform quality. A somewhat similar, but much more careful, investigation was then carried out on $\frac{1}{8}$-inch mild steel plate which had been specially made for experimental purposes but which, unfortunately, had been over-annealed during heat-treatment. The greatest width of bar that can be taken by the 100-ton testing machine in the Engineering Laboratory is 4 inches; so that by using this thickness of plate, the ratio of width to thickness could be increased to nominally 32:1 as compared with 16:1 in the previous work - ratios very much higher, so far as the writer can learn, than have been previously employed except in the case of fabric. To determine whether the results obtained from mild steel were peculiar to that metal or common to other ductile metals which fail with marked local contraction, a third investigation, on lines similar to those adopted immediately above, was conducted on test-pieces cut from $\frac{1}{8}$-inch copper plate also obtained locally and of unknown origin.

The results of the first investigation were communicated to the Royal Society of Edinburgh (1) and published in the Transactions thereof, and those of more practical interest derived from all three enquiries were presented in a paper read before the British Association (2) and reported verbatim in <u>Engineering</u>. The writer's contributions to these joint papers are embodied in, and form the basis of, this Thesis.

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3. Earlier Investigations.

The first record of experimental work on the subject appears to be that of Barba (3). From a 10-mm (0.394-inch) mild steel plate this elastician cut a series of test-bars 10, 20, 30, --- 80 mm. wide: these he tested to destruction, obtaining the results given in Table I. The figures indicate that the yield-point and the tenacity of the metal are not appreciably affected by increase in width of the specimen, the variation in the values being not greater than would normally be expected from the same number of bars (even if of constant width) prepared from a plate taken at random; that is, the differences may readily be ascribed to unavoidable variations of material and errors in testing. On the other hand, the apparent ductility, as measured by the percentage of elongation on gauge-lengths of 50 mm. and 100 mm., shows considerable variation. In Fig. 1 the extension on each gauge-length is plotted against the ratio $\frac{\text{width}}{\text{thickness}}$. It is observed that from a width equal to the thickness up to a width equal to six thicknesses both curves exhibit a more or less steady rise in the value of the extension and at the latter width a well-marked maximum. Barba himself could not account satisfactorily for this maximum. Referring to the divergence

TABLE I.

Tensile Tests of Soft Steel Bars 10 mm. Thick (Barba).

Width mm.	Ratio, $\frac{\text{width}}{\text{thickness}}$.	Yield-point.		Tenacity.		Extension per cent. on	
		Kg. per sq. mm.	Tons per sq. inch.	Kg. per sq. mm.	Tons per sq. inch.	50 mm.	100 mm.
10	1	24.8	15.8	38.4	24.4	37.6	31.0
20	2	24.6	15.6	40.1	25.5	45.0	34.0
30	3	25.4	16.1	39.4	25.0	48.0	35.0
40	4	25.0	15.9	39.8	25.3	52.0	37.2
50	5	24.6	15.6	38.1	24.2	56.0	39.0
60	6	24.9	15.8	37.7	23.9	61.0	40.8
70	7	24.8	15.8	37.8	24.0	57.0	38.5
80	8	23.5	14.9	38.4	24.4	52.0	34.5

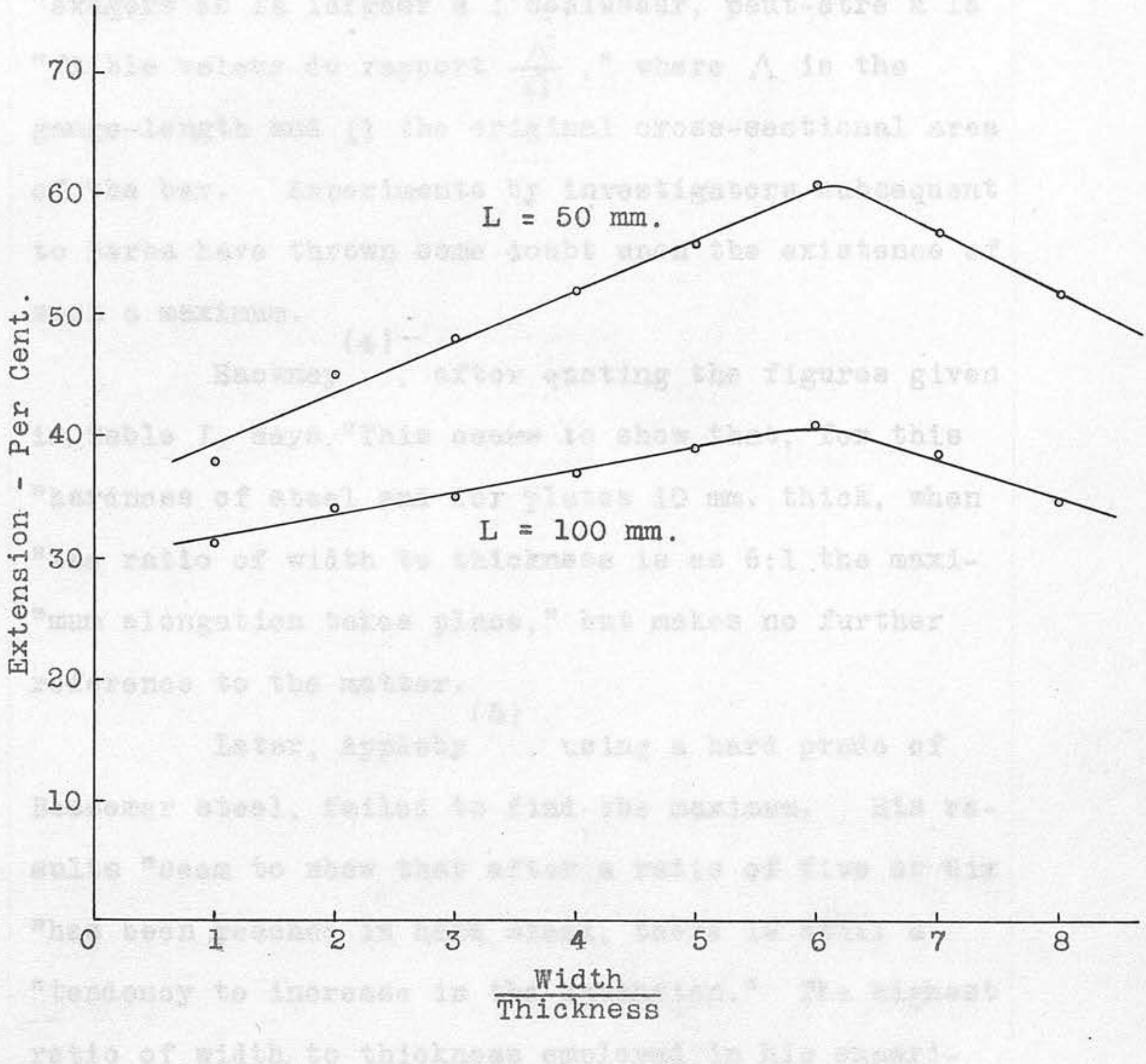
70
60
50
40
30
20
10
0
L = 50 mm.
L = 100 mm.
Extension - Per Cent.
1
2
3
4
5
6
7
8
Width
Thickness

Fig.1.

of the seventh and eighth points from the straight line he says, "On ne sait pas bien encore à quoi "l'attribuer; peut-être est-elle due au rapport "exagéré de la largeur à l'épaisseur, peut-être à la "faible valeur du rapport $\frac{\Lambda^2}{\Omega}$," where Λ is the gauge-length and Ω the original cross-sectional area of the bar. Experiments by investigators subsequent to Barba have thrown some doubt upon the existence of such a maximum.

Hackney (4), after quoting the figures given in Table I, says, "This seems to show that, for this "hardness of steel and for plates 10 mm. thick, when "the ratio of width to thickness is as 6:1 the maxi- "mum elongation takes place," but makes no further reference to the matter.

Later, Appleby (5), using a hard grade of Bessemer steel, failed to find the maximum. His results "seem to show that after a ratio of five or six "has been reached in hard steel, there is still a "tendency to increase in the extension." The highest ratio of width to thickness employed in his experiments was, like that of Barba, eight.

In Unwin's classical paper (6) on tensile tests of mild steel there is no mention whatever of the critical ratio of Barba; in fact, in the correspondence on the paper it was observed that as Barba had found so definitely that a rectangular test-piece gave a greater elongation when six thicknesses wide than when wider or narrower than this, it was strange that neither the author of the paper nor Appleby had been able to confirm this result. The extensions on a constant gauge-length of 8 inches obtained by Unwin from tests of ship and boiler plates frequently show ill-defined maxima which vary in position as regards the ratio of width to thickness. Moreover, in most of his series of test-bars the thickness was varied - a decided disadvantage from the standpoint of the present investigation, because with different thicknesses of plate it is difficult to maintain uniformity of quality. In the same paper Unwin introduces his elongation equation - a linear relation between the percentage of elongation and the ratio of the square root of the cross-sectional area of the bar to the gauge-length. This equation appears to have been derived previously and independently by Barba [7] and by Martens [8]. If _e_ is the percentage of elongation on a gauge-length _L_, obtained from a bar of cross-sectional area _A_, then

$$e = \frac{c\sqrt{A}}{L} + b$$

where the first term on the right is the percentage of elongation due to local contraction and the second the percentage of general elongation. For any given quality of the same material c and b are nearly constant. The factor 100 is merged in the constants. This equation obviously indicates no maximum with increasing width of bar; and experiment shows that it is satisfied only in the case of bars of compact section, such as circular, square, and rectangular of width not greater than about four or five thicknesses.

Modern British writers on the strength of materials seldom mention Barba's maximum; and Continental authorities such as Wève (9) and Bach (10), although referring to the critical ratio, offer no comment thereon.

The writer has also looked into the available work of Denny, Martens, Bach, Rudeloff and others with much profit, but without discovering any treatment of the subject on the lines set forth below.

As the question of Barba's critical ratio was the original starting point of the research, a considerable amount of réference has been made to it in the above short historical sketch.

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4. Experimental Details.

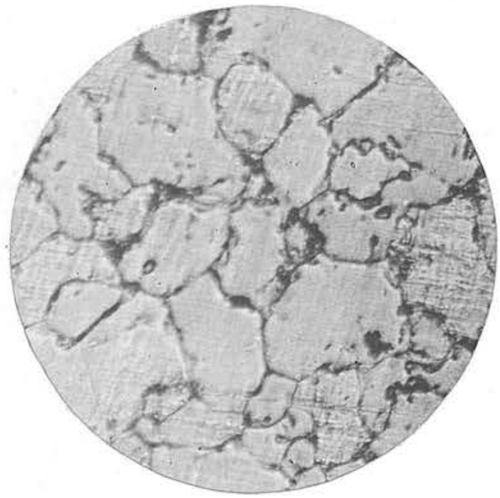
The $\frac{1}{4}$-inch steel and the $\frac{1}{8}$-inch copper were obtained from Messrs John Greig & Sons, Engineers, Edinburgh, and the $\frac{1}{8}$-inch steel direct from Messrs David Colville & Sons, Steelmakers, Motherwell. To determine the direction of rolling of the steel plates, small portions were cut out, roughly polished, and etched with iodine. Thereafter, systematic micro-analyses of all three metals were made: the photographs of Figs. 2, 3, and 4 show the results.

The $\frac{1}{4}$-inch material (Fig. 2) indicates the usual features of soft steel - irregular polygons of ferrite or alpha iron intermingled with darker areas of pearlite or Fe-Fe_3C eutectoid and a few small slag inclusions. The carbon content manifestly lies between 0.12 and 0.15 per cent.

The chemical analysis supplied by Messrs Colville gave 0.13 per cent. of carbon for the $\frac{1}{8}$-inch steel. This plate was rolled from a slab cut from the lower half of a 5-ton ingot of acid Siemen's steel, and was annealed with some thicker plates by being heated to 800° C., furnace-cooled back to 600° C., then air-cooled down to atmospheric temperature. Although the annealing temperature was not too high, the closely connected time and mass factors proved

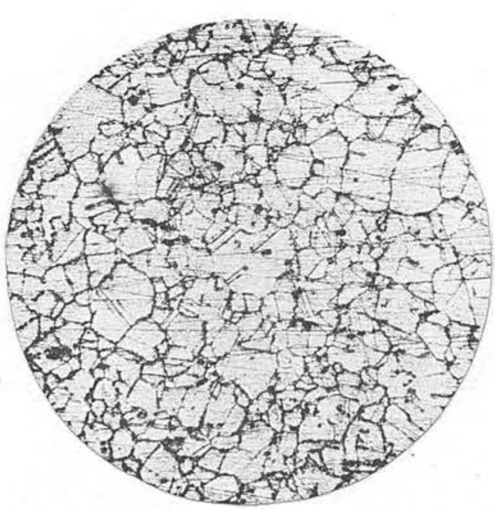
$\frac{1}{4}$-Inch Steel x 150.

Fig. 2.



$\frac{1}{8}$-Inch Steel x 150.

Fig. 3.



$\frac{1}{8}$-Inch Copper x 150.

Fig. 4.

their importance to the slight detriment of the material. Having regard to the magnification, the grain-size revealed by the micrograph, Fig. 3, shows quite conclusively that the plate was over-annealed. Messrs Colville's experience is "that a plate of "this thickness is extremely sensitive to heat treat-"ment and will give results varying within relatively "wide limits." The metal, however, was commercially sound, the quality being that known as "welding boiler."

The copper was practically free from arsenic, and micro-analysis revealed the normal structure (Fig. 4).

The plates were sawn into bars 18 inches long in the direction of rolling and of widths varying from ½ inch to 4 inches by ½-inch intervals: the two thinner plates also yielded bars ¼ inch wide. Three bars of each width were obtained from the ¼-inch plate and four from each of the ⅛-inch plates. Bars of the same width were clamped together and machined on the edges; and the mill scale on the faces was removed by light grinding on the sandstone and rubbing the resulting surfaces with medium emery cloth. By this procedure a fairly uniform section was obtained.

The width and thickness were determined by gauge to 0.001 inch, the maximum variation in dimension in no case exceeding 0.5 per cent. These dimensions, averaged for each set of three bars of the thicker steel and for each set of four bars of the thinner steel and the copper, together with various necessary values calculated therefrom, are given in Tables II, III, and IV. Each result recorded below is accordingly the average obtained from independent tests of three or of four specimens as the case may be.

The centre line on one face of each bar was divided for its entire exposed length of 12 inches into 1-inch lengths in the earlier investigation and into $\frac{1}{2}$-inch lengths in the two later enquiries. A scribing board and steel trammel, made by the writer and shown in Fig. 5, was used for this purpose and gave excellent results. The extension of bars breaking at some distance from the centre of their length could thus be computed as for fracture at that point. It may be mentioned at this stage that bars which broke near the grips or which developed two waists before rupture were rejected and replaced.

The bars cut from the $\frac{1}{4}$-inch plate were marked off on the other face in sub-multiples of the German standard gauge-length 11.3 $\sqrt{\text{area of section}}$, 11.3 being the constant for both cylindrical and flat test-bars with a datum length of 20 cm. and a section

TABLE II.

$\frac{1}{4}$-Inch Steel - Dimensions of Test-Bars.

Nominal Width. Inches.	Actual Width. Inches.	Thickness. Inch.	Area. Sq.inches.	$\sqrt{\text{Area}}$.	$\frac{\text{Width}}{\text{Thickness}}$.
$\frac{1}{2}$	0.450	0.255	0.115	0.339	1.76
1	0.967	0.254	0.245	0.495	3.81
$1\frac{1}{2}$	1.443	0.256	0.370	0.608	5.64
2	1.947	0.257	0.501	0.708	7.57
$2\frac{1}{2}$	2.478	0.259	0.642	0.801	9.57
3	2.897	0.260	0.752	0.867	11.14
$3\frac{1}{2}$	3.451	0.259	0.894	0.945	13.32
4	3.955	0.257	1.016	1.008	15.39

TABLE III.

$\frac{1}{8}$-Inch Steel - Dimensions of Test-Bars.

Nominal Width. Inches.	Actual Width. Inches.	Thickness. Inch.	Area. Sq.Inch.	$\sqrt{\text{Area}}$.	$\frac{\text{Width}}{\text{Thickness}}$.
$\frac{1}{4}$	0.246	0.136	0.033	0.183	1.81
$\frac{1}{2}$	0.494	0.135	0.067	0.258	3.66
1	1.000	0.136	0.136	0.369	7.35
$1\frac{1}{2}$	1.494	0.136	0.203	0.451	10.98
2	2.007	0.137	0.275	0.524	14.65
$2\frac{1}{2}$	2.500	0.137	0.342	0.585	18.25
3	2.991	0.136	0.407	0.638	21.99
$3\frac{1}{2}$	3.505	0.135	0.473	0.688	25.96
4	4.006	0.134	0.537	0.733	29.89

TABLE IV.

$\frac{1}{8}$-Inch Copper - Dimensions of Test-Bars.

Nominal Width. Inches.	Actual Width. Inches.	Thickness. Inch.	Area. Sq.Inch.	$\sqrt{\text{Area}}$.	$\frac{\text{Width}}{\text{Thickness}}$.
$\frac{1}{4}$	0.251	0.132	0.033	0.182	1.90
$\frac{1}{2}$	0.489	0.130	0.064	0.252	3.76
1	1.000	0.130	0.130	0.360	7.69
$1\frac{1}{2}$	1.486	0.132	0.196	0.443	11.26
2	2.000	0.133	0.266	0.516	15.04
$2\frac{1}{2}$	2.496	0.135	0.337	0.580	18.49
3	2.994	0.134	0.401	0.633	22.34
$3\frac{1}{2}$	3.488	0.133	0.464	0.681	26.22
4	3.997	0.134	0.536	0.732	29.83

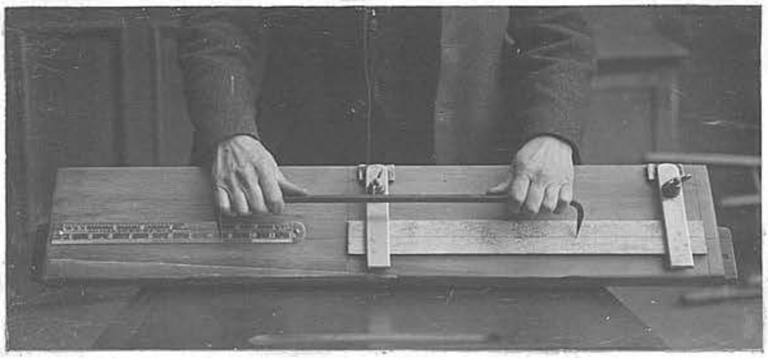


Fig.5.

of 3.14 sq. cm. Within the limits of the sectional dimensions used in practice, flats marked off in terms of such a gauge-length may be regarded as virtually similar bars. As the extensions obtained by direct measurement on this face were found to be practically identical with those determined from large-scale extension-length curves plotted from the elongations on successive inches on the other face, this procedure was not employed in the experiments on the thinner plates.

The utmost care was taken to secure axial application of the load; and, in order to secure uniformity in the rate of its application, the same straining speed was employed throughout the tests - about 0.12 inch per minute or 1 per cent. per minute on the 12-inch length between the grips.

The load at yielding for both steels was determined quite readily by the steelyard drop; but, in the case of the copper, recourse was had to autographic diagrams as a check, the drop not being strongly marked. The maximum load likewise was obtained with accuracy. Owing to the rapidity with which such ductile metals draw out in the last phase of the test, it is somewhat difficult to determine by hand-working of the jockey-weight the exact load at which rupture occurs. This difficulty was naturally more marked in the case of the thicker steel than in those of the thinner metals. The stress calculated from this final load and the minimum cross-sectional area of the broken bar is termed, in what follows, the <u>mean breaking stress.</u>

From these loads and initial and final measurements made on the bars, the variation in the mechanical properties corresponding to change in the width, or more correctly, to change in the ratio of width to thickness, have been determined. The results recorded below may be taken as representing the product, within reasonable limits, of considerable care and precision, and are discussed most conveniently under the following heads: -

(1) The yield-point, the tenacity, and the mean breaking stress;

(2) The extension on fixed lengths of the bar;

(3) The extension on lengths proportional to the square root of the cross-sectional area of the bar;

(4) The extension on lengths proportional to the width of the bar;

(5) The extension on lengths proportional to the ratio of the cross-sectional area to the perimeter of the bar;

(6) The reduction of area at fracture;

(7) The ratio $\frac{\text{width}}{\text{thickness}}$ - initial and final.

In the concluding Article the whole is briefly summarised.

5. Yield-Point, Tenacity, and Mean Breaking Stress.

The figures of Table V show the effect of variation of width of specimen on these properties of the metals.

The yield-point remains practically constant over the entire range of observation in each case, the average values being 18.14 tons per square inch for the $\frac{1}{4}$-inch steel, 12.82 tons per square inch for the $\frac{1}{8}$-inch steel, and 7.32 tons per square inch for the $\frac{1}{8}$-inch copper. The width of the specimen therefore appears to have no appreciable influence upon the elastic strength of mild steel or rolled copper.

The tenacity or commercial maximum stress also appears to be but little affected. In the case of the two steels there is a slight tendency to rise as the width is increased, but this effect is noticeably absent in the case of the copper. The average values are 25.48 tons per square inch for the thicker steel, 22.37 tons per square inch for the thinner steel, and 14.52 tons per square inch for the copper.

The ratio $\frac{\text{yield-point}}{\text{tenacity}}$ may be taken as an indication of the degree of mechanical work to which material has been subjected prior to being tested:

TABLE V.

Yield Point, Tenacity, and Mean Breaking Stress.

Nominal Width	¼-Inch Steel.			⅛-Inch Steel.			⅛-Inch Copper.		
	Yield Point.	Tenacity.	Mean Breaking Stress.	Yield Point.	Tenacity.	Mean Breaking Stress.	Yield Point.	Tenacity.	Mean Breaking Stress.
Inches.	Tons per sq.inch.	Tons per sq.inch.	Tons per sq.inch.	Tons per sq.inch.	Tons per sq.inch.	Tons per sq.inch.	Tons per sq.inch.	Tons per sq.inch.	Tons per sq.inch.
¼	-	-	-	12.89	22.09	46.4	7.32	14.55	26.3
½	18.17	25.26	54.3	12.76	22.19	45.5	7.49	14.47	26.1
1	18.28	25.32	50.6	12.74	22.22	46.6	7.20	14.60	26.4
1½	18.29	25.45	48.8	12.70	22.26	45.9	7.22	14.58	25.7
2	18.10	25.45	46.0	12.76	22.47	46.0	7.30	14.54	24.9
2½	18.08	25.47	49.9	12.87	22.53	45.7	7.26	14.38	25.6
3	18.00	25.40	47.9	12.95	22.53	47.4	7.38	14.48	25.6
3½	18.05	25.66	50.0	12.90	22.46	47.2	7.41	14.56	26.3
4	18.16	25.87	50.7	12.82	22.55	46.8	7.35	14.50	25.5

its reciprocal has been used as a measure of toughness[(11)]. Using the figures given immediately above, the average values of this function are 0.71 for the $\frac{1}{4}$-inch steel, 0.57 for the $\frac{1}{8}$-inch steel, and 0.50 for the $\frac{1}{8}$-inch copper. The thicker steel appears to have received little, if any, annealing, whilst the thinner steel has been too well treated in this respect. The copper was stated to be annealed, and the value of the ratio in this case would suggest that it is so.

The determination of the factors of the mean breaking stress does not admit of great accuracy, but much care was taken to secure results as correct as possible. As mentioned above, the exact measurement of the load at rupture is in ordinary circumstances a matter of some difficulty. Further, the fractured surface of an initially rectangular section is not a rectangle, but has curved sides. These do not always occur symmetrically about the centre of the width, but are usually so with respect to the centre line of the thickness: the ultimate shape is doubtless determined to a considerable extent by the presence of small local defects. In order to secure reliable results, the two parts of the broken bar were fitted together under gentle end pressure, and the minimum width determined by averaging four readings -

two with each face up. The mean thickness was found from twelve readings - six from each portion of the bar, the centre reading in every case being doubled. The values of the mean breaking stress for the $\frac{1}{4}$-inch steel vary considerably, but those for the $\frac{1}{8}$-inch steel and the copper deviate little from constancy and, accordingly, are much more satisfactory: the irregularities in the first case are doubtless due to uncertainty in the determination of the actual load at the moment of rupture. This property, although of little use to the engineer, furnishes information regarding the true maximum stress sustained by the material. The average values for the three series of tests are 49.8, 46.4, and 25.8 tons per square inch respectively.

The figures of Table V, like those of Table I (Barba), cannot be said to support the generally accepted view that the wider the test-bar, the lower the strength both elastic and ultimate. With well-conditioned specimens of sound material, axially loaded through first-class grips in a good machine, there appears to be *a priori* no reason why the stress should not be uniformly distributed over the wider specimens; but in circumstances other than these, the above-mentioned opinion is doubtless found to be correct - for example, in ordinary material the probability of serious defect increases with the section, and unevenly distributed pressure of the grips induces bending action. The causes of any decline in the values are extrinsic or mechanical rather than intrinsic or geometrical.

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6. Extension on Fixed Lengths.

The measurements required under this head were taken along the scribed centre line on that face of the bar which had been originally marked off in 1-inch or $\frac{1}{2}$-inch lengths, and in every case were made symmetrically about the inch or half-inch within which fracture occurred, the missing spaces of the shorter portion of the bar being obtained, where necessary, by doubling the measurements made on the available datum inches or half-inches of the longer portion and adding these in order to the previous totals. This procedure, which should always be employed, is necessary in testing for comparative purposes: it nullifies to a great extent the effect of variation in the position of the fracture: incidentally, it favours the manufacturer. The measurement of the actual stretched length was made by using a steel rule, reading to 0.01 inch, in conjunction with a watchmaker's glass. By this means it was quite an easy matter to read to 0.005 inch - a close enough approximation for permanently deformed material. To obtain the nett elongation along the centre line, the breadth of the gap at the fracture was deducted from these overall dimensions. Owing to the fact that rupture generally begins in or about the axis of a bar and progresses out towards metal which is still stretching, the breadth of the gap is usually a maximum at or near the centre of the width of a rectangular test-piece, and for metal of uniform thickness is roughly proportional to the width of the bar.

From the measurements of the nett stretch the mean extensions on fixed lengths of 1, 3, 5,---15 inches for each set of the thicker steel bars and on fixed lengths of 1, 2, 3,---15 inches for each set of the thinner steel and copper bars were computed: these are set forth in Tables VI, VII, and VIII. The extension on the first inch is subject to error since the variation of position of the fracture within this inch has been neglected. This error increases in importance with width of bar, but diminishes rapidly with increase of gauge-length: in the widest bars its effect was found to be negligible in lengths over 4 inches.

Irregularity in value of the extensions of any bar, due to lack of uniformity in the distribution of elongation in the portions outside the reduced region, was to a great extent modified on taking the average for the set of bars. This irregularity, commonly attributed to variation of hardness, is more probably due to variation in the average orientation of the crystals, deformation by slips being thus more

TABLE VI.

$\frac{1}{4}$-Inch Steel - Extension on Fixed Lengths.

Nominal Width. Inches.	Extension per cent. on a Length of							
	1	3	5	7	9	11	13	15 inches.
$\frac{1}{2}$	45.0	30.2	25.3	23.8	22.2	20.9	19.9	19.1
1	50.0	34.2	28.8	26.1	24.7	23.8	22.8	22.2
$1\frac{1}{2}$	55.0	38.9	33.0	29.4	27.1	25.5	24.3	23.3
2	59.7	41.7	34.7	31.0	28.2	26.5	25.2	24.0
$2\frac{1}{2}$	65.0	43.2	35.3	31.4	28.6	26.8	25.3	24.2
3	62.3	42.8	35.1	31.2	28.2	26.2	24.6	23.4
$3\frac{1}{2}$	67.0	44.8	36.5	32.4	29.3	27.0	25.3	24.0
4	68.0	47.3	38.6	34.4	31.2	29.1	27.1	25.7

TABLE VII.

$\frac{1}{8}$-Inch Steel - Extension on Fixed Lengths.

Nominal Width. Inches.	Extension per cent. on a Length of														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15 inches
$\frac{1}{4}$	36.5	30.1	27.3	25.5	24.3	23.3	22.9	22.4	22.1	21.8	21.6	21.3	21.1	20.8	20.6
$\frac{1}{2}$	41.8	33.9	30.5	28.4	27.1	26.1	25.3	24.6	23.8	23.2	22.7	22.3	22.0	21.7	21.4
1	47.6	38.4	34.0	31.5	29.9	28.5	27.4	26.6	25.9	25.2	24.6	24.2	23.8	23.5	23.1
$1\frac{1}{2}$	49.4	39.1	34.1	31.4	29.5	28.3	27.3	26.4	25.7	25.0	24.4	23.9	23.4	23.0	22.5
2	52.1	41.4	36.7	33.6	31.8	30.2	29.0	27.9	27.0	26.3	25.7	25.2	24.7	24.2	23.8
$2\frac{1}{2}$	50.5	40.9	36.3	33.4	31.7	30.3	29.3	28.3	27.2	26.4	25.8	25.2	24.6	24.1	23.5
3	51.7	42.7	38.2	35.4	33.1	31.4	30.2	29.1	28.2	27.4	26.7	26.1	25.6	25.0	24.5
$3\frac{1}{2}$	51.9	42.6	37.8	34.5	32.1	30.4	29.1	28.1	27.3	26.6	25.9	25.3	24.7	24.1	23.7
4	54.7	44.9	39.3	35.7	33.4	31.8	30.6	29.7	29.2	28.4	27.7	27.1	26.5	26.0	25.5

TABLE VIII.

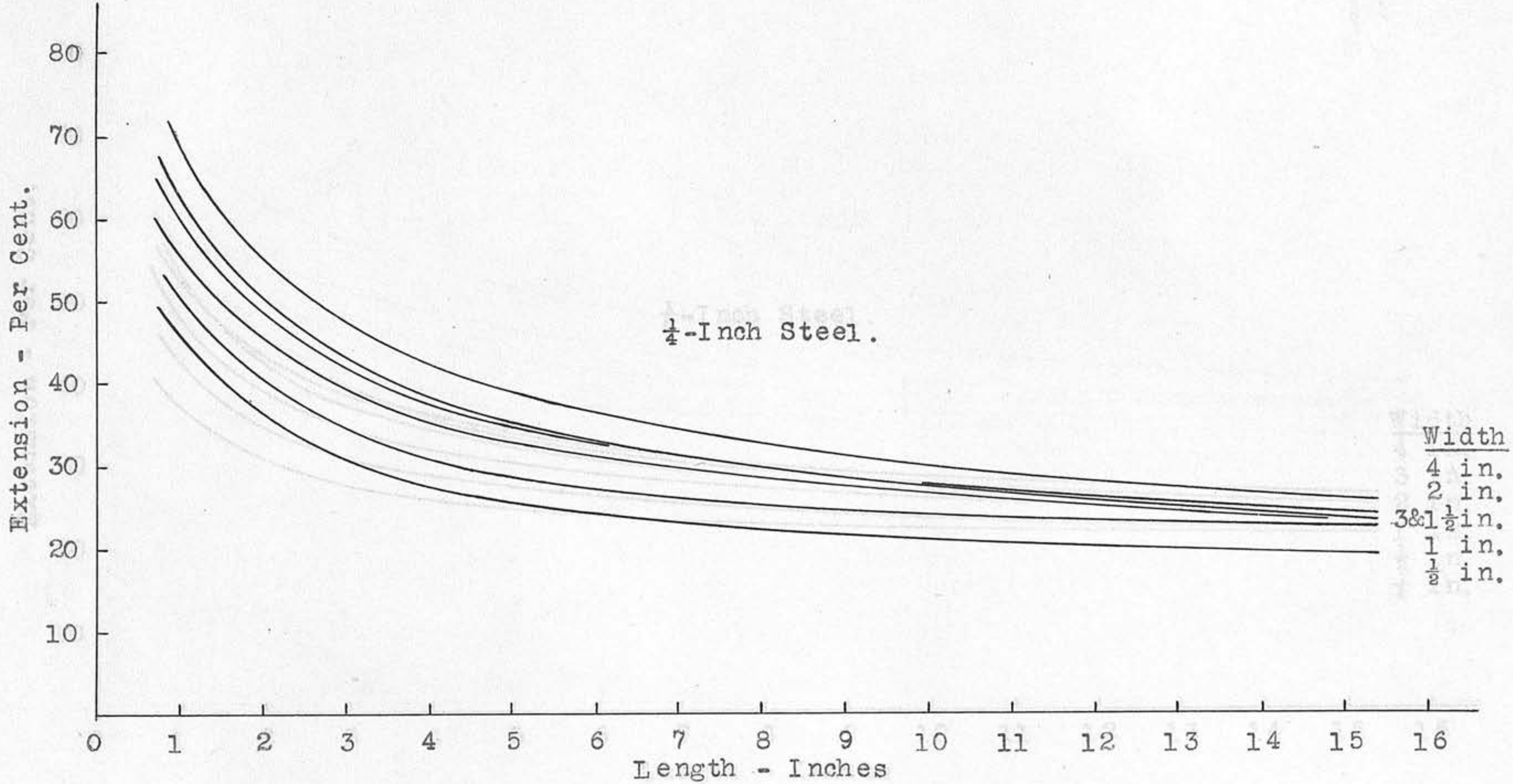
$\frac{1}{8}$-Inch Copper - Extension on Fixed Lengths.

Nominal Width. Inches.	Extension per cent. on a Length of														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15 inches.
$\frac{1}{4}$	47.4	42.4	40.2	38.2	36.6	35.2	33.9	32.9	32.3	31.7	31.3	30.9	30.6	30.3	30.0
$\frac{1}{2}$	52.9	45.0	40.9	39.0	37.7	36.7	35.9	35.4	34.8	34.6	34.2	33.9	33.6	33.2	32.9
1	60.4	50.2	45.1	42.1	40.0	38.7	37.6	37.2	36.9	36.6	36.2	35.8	35.4	35.1	34.7
$1\frac{1}{2}$	63.1	52.9	47.5	44.1	41.9	40.3	39.2	38.3	37.7	37.3	36.9	36.4	36.0	35.6	35.2
2	56.5	50.2	45.7	42.6	40.7	39.1	37.9	37.2	36.9	36.6	36.2	35.8	35.3	34.7	34.2
$2\frac{1}{2}$	58.5	52.1	47.8	44.9	42.9	41.3	40.2	39.2	38.4	38.0	37.7	37.5	37.3	36.9	36.6
3	60.4	55.0	50.8	47.9	46.1	44.8	44.0	43.5	43.1	42.6	42.0	41.2	40.6	39.9	39.4
$3\frac{1}{2}$	68.1	61.5	57.2	53.8	51.2	48.9	47.2	45.7	44.6	43.5	42.7	41.7	40.9	40.3	39.6
4	58.8	54.6	51.2	48.1	45.8	44.0	42.7	41.6	40.7	39.9	39.2	38.6	38.0	37.5	37.0

easily effected in some parts than in others. In this connection the writer would say that he has not infrequently seen the edges of a wide flat bar become distinctly corrugated just before the maximum load was reached.

The nett extensions were then plotted, separately for each width of bar, to a large scale against the length, the resulting curves being of great value, for from them could be found for a given width the extension on any datum length, fixed or variable, from 1 inch to 15 inches. For example, as stated in Art. 4, the extensions on gauge-lengths equal to 11.3 $\sqrt{\text{area}}$ measured from these curves for the thicker steel were found to be practically identical with those determined by direct measurement on the other face which had been marked off in sub-multiples of that function.

The form of these extension-length curves is indicated in Figs. 6, 7, and 8 which for the sake of clarity of diagram give in each case only a selection of the results. The curves are of the usual hyperbolic type, showing that with increase of gauge-length the extension falls rapidly at first, declines more slowly, and finally tends towards a minimum which is probably constant for all widths of specimen cut from the same plate. The curve for any set of bars does



¼-Inch Steel.
Extension - Per Cent.
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
Length - Inches
Width
4 in.
2 in.
3&1½in.
1 in.
½ in.

Fig. 6.

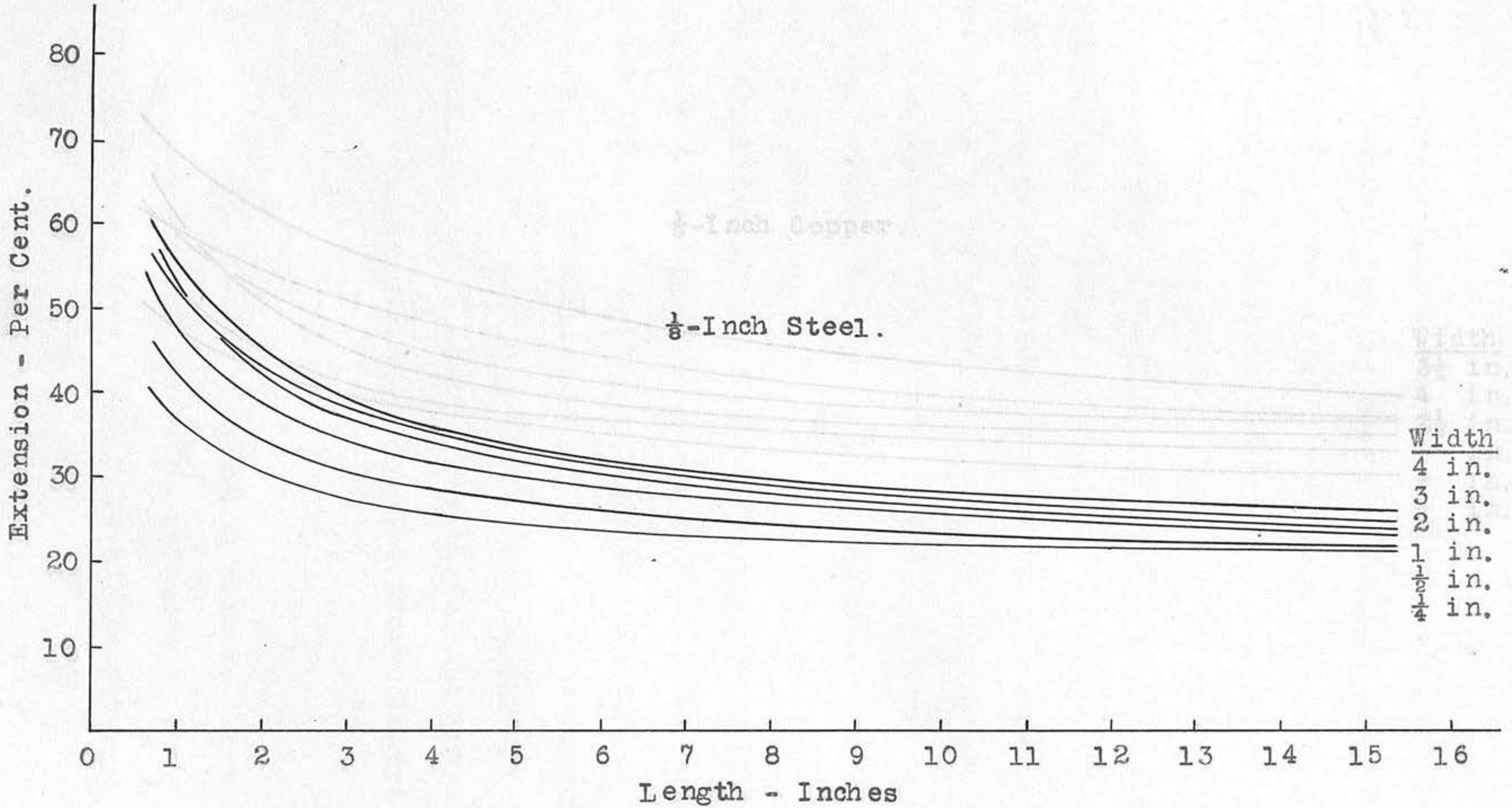
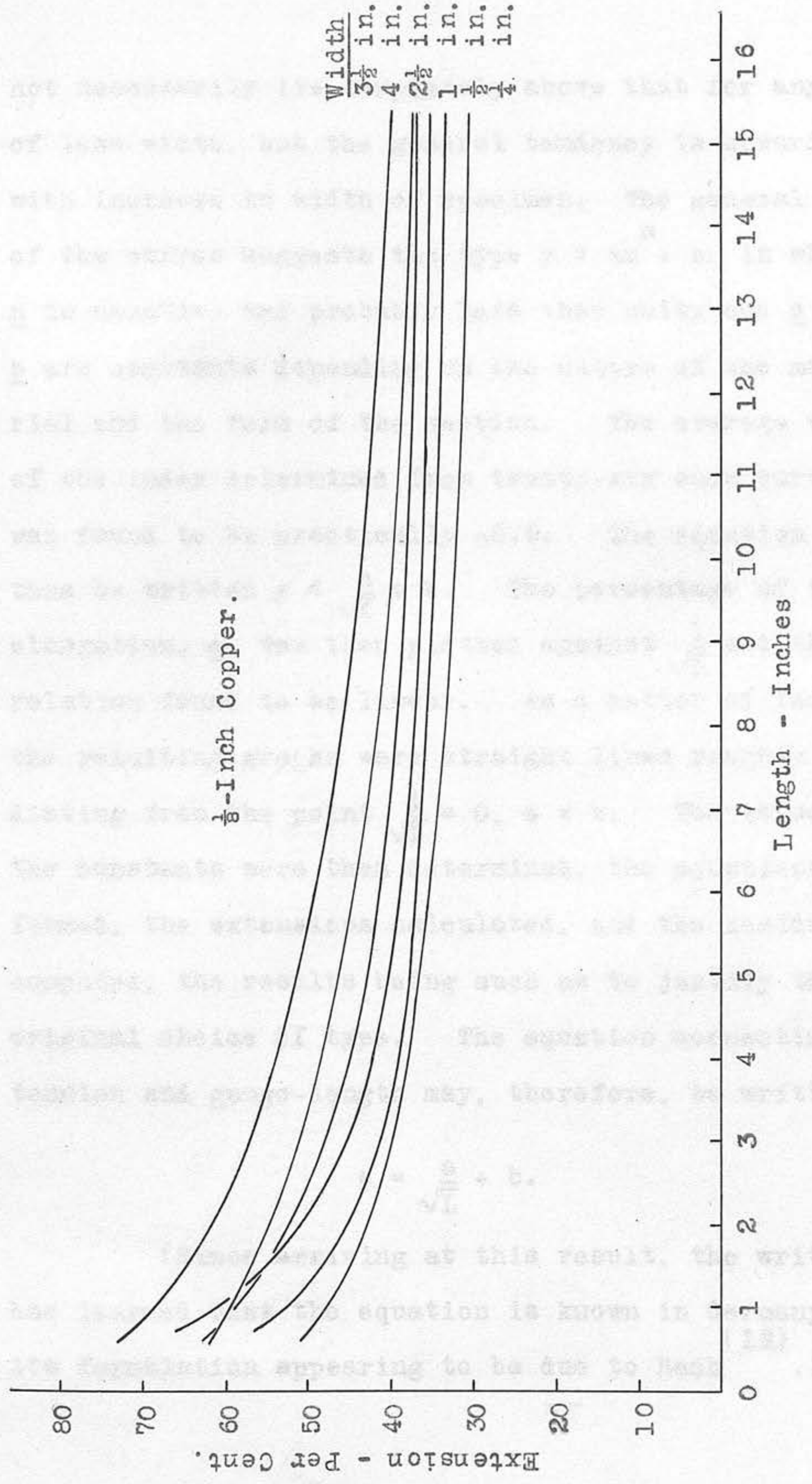

80
70
60
50
40
30
20
10
Extension - Per Cent.
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
Length - Inches
⅛-Inch Steel.
Width
4 in.
3 in.
2 in.
1 in.
½ in.
¼ in.

Fig.7.



1/8-Inch Copper.
Width
3½ in.
4 in.
2½ in.
1 in.
½ in.
¼ in.
Extension - Per Cent.
80
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
Length - Inches

Fig. 8.

not necessarily lie completely above that for any set of less width, but the general tendency is upwards with increase in width of specimen. The general form of the curves suggests the type $y = ax^n + b$, in which n is negative and probably less than unity and a and b are constants depending on the nature of the material and the form of the section. The average value of the index determined from twenty-six such curves was found to be practically -0.5. The equation may thus be written $y = \frac{a}{\sqrt{x}} + b$. The percentage of total elongation, e, was then plotted against $\frac{1}{\sqrt{L}}$ and the relation found to be linear. As a matter of fact, the resulting graphs were straight lines roughly radiating from the point $\frac{1}{\sqrt{L}} = 0$, $e = b$. The values of the constants were then determined, the equations formed, the extensions calculated, and the residuals computed, the results being such as to justify the original choice of type. The equation connecting extension and gauge-length may, therefore, be written

$$e = \frac{a}{\sqrt{L}} + b.$$

(Since arriving at this result, the writer has learned that the equation is known in Germany, its formulation appearing to be due to Bach (12).)

In experimental work on the Strength of Materials, other than fine elastical researches, the amount of labour involved in calculating the values of empirical constants by the Method of Least Squares is quite out of proportion to the benefit, if any, derived so far as accuracy is concerned. Accordingly, throughout this Thesis the Method of Averages has been freely employed. This procedure applied to the figures of Tables VI, VII, and VIII gives in each case comparatively close values of b and values of a varying more widely and somewhat irregularly with the width of specimen. For the sets $1\frac{1}{2}$ inches and 4 inches wide - bars of moderate and extreme width, the one 1 inch below and the other $1\frac{1}{2}$ inches above the B.E.S.A. maximum allowable width for $\frac{1}{4}$-inch and $\frac{1}{8}$-inch plates - the equations obtained are as follows: -

$\frac{1}{4}$-Inch Steel.

Width = $1\frac{1}{2}$ inches, $e = \dfrac{49.87}{\sqrt{L}} + 10.47$

Width = 4 inches, $e = \dfrac{68.19}{\sqrt{L}} + 8.30$

$\frac{1}{8}$-Inch Steel.

Width = $1\frac{1}{2}$ inches, $e = \dfrac{36.83}{\sqrt{L}} + 13.20$

Width = 4 inches, $e = \dfrac{42.20}{\sqrt{L}} + 14.82$

$\frac{1}{8}$-Inch Copper.

Width = $1\frac{1}{2}$ inches, $e = \dfrac{35.66}{\sqrt{L}} + 26.09$

Width = 4 inches, $e = \dfrac{45.09}{\sqrt{L}} + 25.53.$

These expressions were used in calculating the values of the extensions as set forth in Tables VIA, VIIA, and VIIIA, which for brevity give only the results obtained on the odd inches. Apart from the figures for the first inch in the contracted zone, the calculated extensions are in quite good practical agreement with the observed values, in point of fact, the residuals are in general smaller than the differences existing between the individual bars of a set. The larger differences for the first inch are mainly due to the error referred to above, an error inevitably present when rupture occurs at a point other than the middle of the inch. In any case, the use of the first inch as a commercial gauge-length is inadmissible.

The equation $e = b$ obviously represents the horizontal asymptote of curves such as those under consideration, and from theoretical considerations b would appear to be the true value of the general extension or the constant minimum towards which the experimental curves tend, the local being a negligible

TABLE VI A.

$\frac{1}{4}$-Inch Steel - Calculated Values of Extension on Fixed Lengths.

Length.	Width = $1\frac{1}{2}$ inches.			Width = 4 inches.		
	Extension per cent.		Difference.	Extension per cent.		Difference.
Inches.	Observed.	Calculated		Observed.	Calculated.	
1	55.0	60.3	-5.3	68.0	76.5	-8.5
3	38.9	39.2	-0.3	47.3	47.6	-0.3
5	33.0	32.8	+0.2	38.6	38.8	-0.2
7	29.4	29.3	+0.1	34.4	34.1	+0.3
9	27.1	27.1	±0.0	31.2	31.0	+0.2
11	25.5	25.5	±0.0	29.1	28.8	+0.3
13	24.3	24.3	±0.0	27.1	27.2	-0.1
15	23.3	23.3	±0.0	25.7	25.9	-0.2

TABLE VII A.

$\frac{1}{8}$-Inch Steel - Calculated Values of Extension on Fixed Lengths.

Length. Inches.	Width = $1\frac{1}{2}$ inches.			Width = 4 inches.		
	Extension per cent.		Difference.	Extension per cent.		Difference.
	Observed.	Calculated.		Observed.	Calculated.	
1	49.4	50.0	-0.6	54.7	57.0	-2.3
3	34.1	34.4	-0.3	39.3	39.2	+0.1
5	29.5	29.7	-0.2	33.4	33.7	-0.3
7	27.3	27.1	+0.2	30.6	30.8	-0.2
9	25.7	25.5	+0.2	29.2	28.9	+0.3
11	24.4	24.3	+0.1	27.7	27.5	+0.2
13	23.4	23.4	±0.0	26.5	26.5	±0.0
15	22.5	22.7	-0.2	25.5	25.7	-0.2

TABLE VIII A.

$\frac{1}{8}$-Inch Copper - Calculated Values of Extension on Fixed Lengths.

Length.	Width = $1\frac{1}{2}$ inches.			Width = 4 inches.		
	Extension per cent.		Difference.	Extension per cent.		Difference
Inches.	Observed.	Calculated.		Observed.	Calculated.	
1	63.1	61.8	+1.3	58.8	70.6	-11.8
3	47.5	46.7	+0.8	51.2	51.5	- 0.3
5	41.9	42.0	-0.1	45.8	45.7	+ 0.1
7	39.2	39.6	-0.4	42.7	42.6	+ 0.1
9	37.7	38.0	-0.3	40.7	40.5	+ 0.2
11	36.9	36.8	+0.1	39.2	39.1	+ 0.1
13	36.0	36.0	±0.0	38.0	38.0	±0.0
15	35.2	35.3	-0.1	37.0	37.2	-0.2

part of the total extension when the gauge-length is increased indefinitely. Taking the $\frac{1}{8}$-inch steel because of its uniformity of quality, the values of b obtained with increasing width of bar are 15.6, 13.7, 14.1, 13.2, 13.2, 13.0, 13.4, 12.5, and 14.8 per cent. The average of these is 13.7 per cent. - a value of the general extension necessarily lower than that obtained from the more usual equation $e = \frac{a}{L} + b$. This latter expression, invariably given in British writings on the subject (Unwin[13], Morley[14], Batson and Hyde[15], and others) as the law of variation of extension with gauge-length, is derived from simple rational considerations and not directly from the results of experiment. Applying it to the two sets of bars under consideration, the formulae obtained are: -

$$\text{Width} = 1\tfrac{1}{2} \text{ inches,} \quad e = \frac{48.14}{L} + 19.70$$

$$\text{Width} = 4 \text{ inches,} \quad e = \frac{55.15}{L} + 22.27$$

The values of the second constant are about those normally associated in this country with the general extension of mild steel. The extensions calculated from these equations give residuals of -18.4, -1.6, +0.2, +0.7, +0.6, +0.3, ±0.0, and -0.4 for the set $1\frac{1}{2}$ inches wide and -22.7, -1.3, +0.1, +0.4, +0.8, +0.4, ±0.0, and -0.4 for the 4-inch set, the numbers being arranged according to increasing values of gauge-length. These differences are not nearly so satisfactory as those determined from the purely empirical formulae and given in Table VIIA.

The equation $e = \frac{a}{\sqrt{L}} + b$ is that of a curve which fits the observed results with practically sufficient accuracy over the whole experimental range, the plus and minus signs being fairly well distributed; on the other hand, the equation $e = \frac{a}{L} + b$ represents a curve which cuts the experimentally determined graph in two points - in fact, it is a totally different curve, and is utilisable only over a certain range, and in that, not too satisfactorily. These observations apply generally to the twenty-six curves (eight for the $\frac{1}{4}$-inch steel and nine each for the $\frac{1}{8}$-inch steel and the copper) derived from the results listed in Tables VI, VII, and VIII.

The writer is of opinion that the lower is probably the truer value of the general extension. Localisation of elongation in the case of very ductile metals is not instantaneous; at the maximum load, the lever remaining steady, the test-bar stretches both generally and locally, and during this phase there is in operation what may be termed a selective process which ultimately determines the point of final contraction. It is conceivable, therefore, that for such metals the constant b in the ordinarily accepted formula contains more than the true general extension.

The curves for both equations have the e-axis as vertical asymptote - a condition which would mean that the rate of extension in the fracture was infinite. This is, of course, absurd. On the justifiable assumption of perfect plasticity in the contracting zone, the maximum limiting extension, or the extension on an indefinitely small length in the fracture, can be calculated from the well-known expression

$$e = \frac{A - A'}{A'},$$

in which A is the initial and A' the final cross-sectional area of the ruptured bar. The average values of this quantity, so determined, are given in Table IX and shown in Figs. 9, 10, and 11 plotted against the ratio $\frac{\text{width}}{\text{thickness}}$. The curves are of the same form as those given later for the reduction of area, the two measures being closely related.

To determine the equations, the curves were rectified and the Method of Averages employed. For the sake of brevity, the equations and the calculated values of the extension in the fracture for the thinner steel alone are given.

TABLE IX.

Extension in the Fracture.

Nominal Width. Inches.	Extension per cent.		
	$\frac{1}{4}$-Inch Steel.	$\frac{1}{8}$-Inch Steel.	$\frac{1}{8}$-Inch Copper.
$\frac{1}{4}$	-	164	107
$\frac{1}{2}$	179	153	109
1	147	149	107
$1\frac{1}{2}$	128	145	100
2	122	141	93
$2\frac{1}{2}$	128	139	91
3	117	135	89
$3\frac{1}{2}$	127	133	91
4	121	135	86

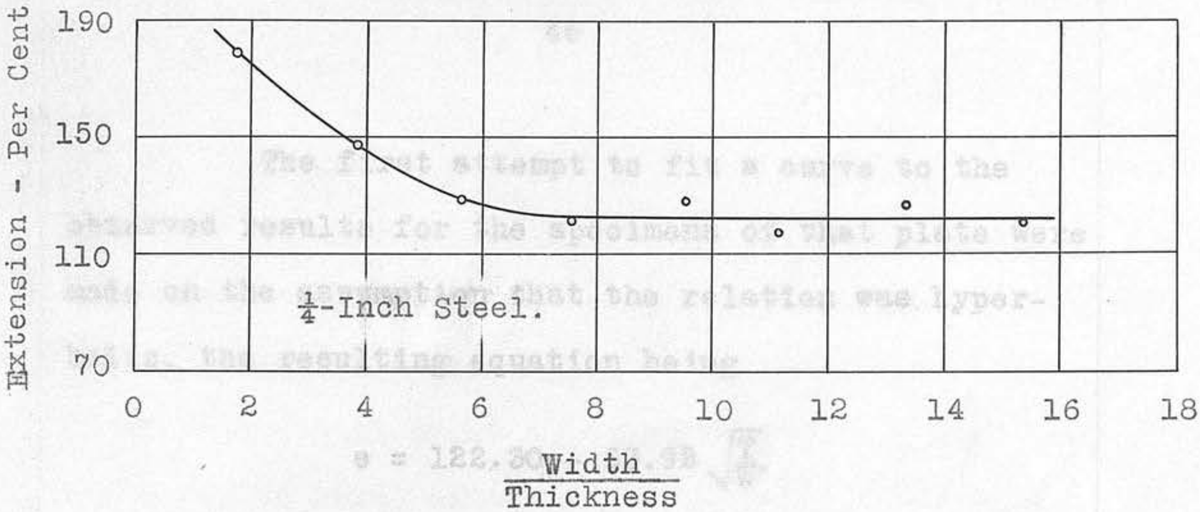


Fig. 9.

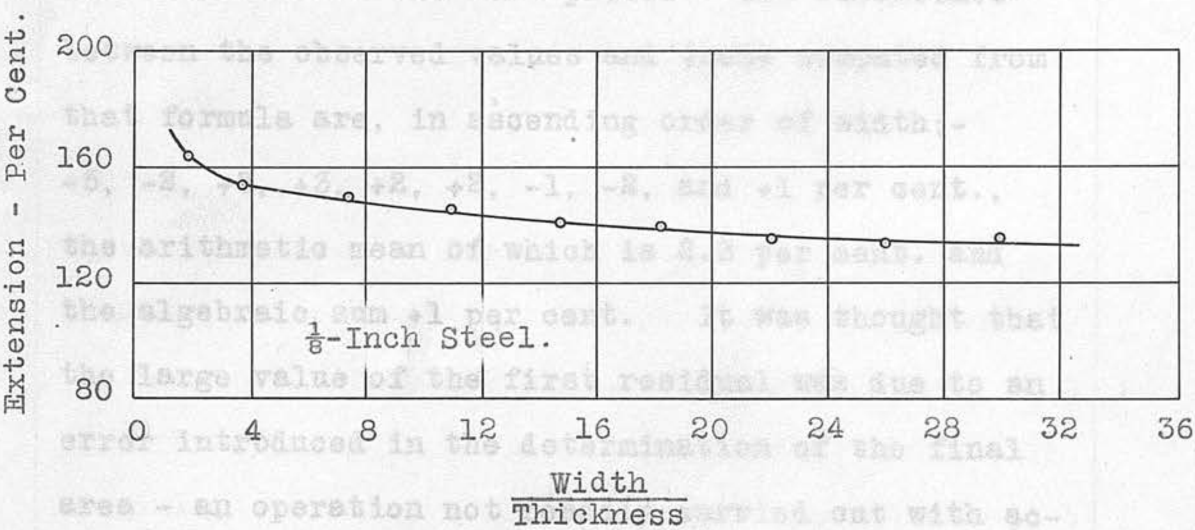


Fig.10.

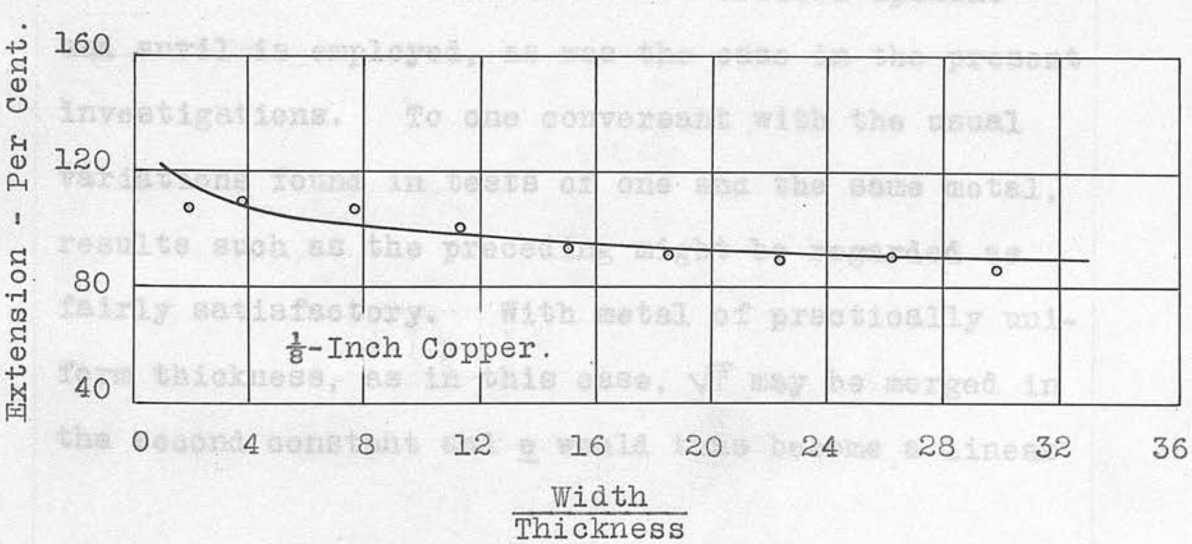


Fig.11.

The first attempt to fit a curve to the observed results for the specimens of that plate were made on the assumption that the relation was hyperbolic, the resulting equation being

$$e = 122.30 + 63.92\sqrt{\frac{T}{W}},$$

in which W and T represent respectively the width and the thickness of the test-piece. The differences between the observed values and those computed from that formula are, in ascending order of width:-
-5, -2, +3, +3, +2, +2, -1, -2, and +1 per cent., the arithmetic mean of which is 2.3̇ per cent. and the algebraic sum +1 per cent. It was thought that the large value of the first residual was due to an error introduced in the determination of the final area - an operation not readily carried out with accuracy on a bar initially $\frac{1}{4}$ inch by $\frac{1}{8}$ inch in section, even when a micrometer with conical spindle and anvil is employed, as was the case in the present investigations. To one conversant with the usual variations found in tests of one and the same metal, results such as the preceding might be regarded as fairly satisfactory. With metal of practically uniform thickness, as in this case, $\sqrt{T}$ may be merged in the second constant and e would thus become a linear

function of $\frac{1}{\sqrt{W}}$.

Further consideration of the experimental data, however, showed that a much better 'fit' was obtained by expressing the limiting extension either as an exponential or as a logarithmic function of the width or, as here, of the ratio $\frac{\text{width}}{\text{thickness}}$, the equations being of the forms:-

$$e = a\epsilon^{b\frac{W}{T}} + c$$

and $$e = a + b \log \frac{W}{T}.$$

In the former, ϵ is Napier's base and c a positive constant representing the asymptotic value of the extension in the fracture or the value below which this property apparently would not fall, however wide the specimen: in both, the constants a are positive and b negative. Employment of the Method of Averages to determine the constants resulted in the formation of the following formulae:-

$$e = 35.29\epsilon^{-0.0606\frac{W}{T}} + 127$$

and $$e = 169.92 - 25.06 \log\frac{W}{T},$$

from which the calculated values of the extension listed in Table IXA were obtained.

TABLE IXA.

$\frac{1}{8}$-Inch Steel - Calculated Values of Extension in the Fracture.

Nominal Width.	Extension per cent. (Exponential)			Extension per cent. (Logarithmic)		
Inches.	Observed.	Calculated.	Difference.	Observed.	Calculated.	Difference.
$\frac{1}{4}$	164	158	+6	164	164	±0
$\frac{1}{2}$	153	155	-2	153	156	-3
1	149	150	-1	149	148	+1
$1\frac{1}{2}$	145	145	±0	145	144	+1
2	141	141	±0	141	141	±0
$2\frac{1}{2}$	139	139	±0	139	138	+1
3	135	136	-1	135	136	-1
$3\frac{1}{2}$	133	134	-1	133	134	-1
4	135	133	+2	135	133	+2

The degree of approximation of the computed to the observed values is seen to be of a much higher order on the later bases than on the previous assumption. The mean value of the exponential residuals, irrespective of sign, is 1.4 per cent. and the algebraic sum +3 per cent., the similar values from the logarithmic differences being 1.1 per cent. and zero respectively. The exponential equation gives a perfect 'fit' about the middle but falls away slightly towards the ends of the range, the high value of the first residual being doubtless due to the divergence of the curves for low values of the argument. On the other hand, the logarithmic relation gives a very good distribution as regards both magnitude and sign over the whole range.

As the merit of a formula to be used in the test house lies not only in its fitting the experimental results with some degree of accuracy but also in its simplicity and immediate applicability, the logarithmic form is to be preferred. It may now with reason be stated that, within the range of the experiments, the relation between the extension in the fracture and the width of specimen is exponential or, if preferred, the percentage of elongation may be regarded simply as a linear function of the logarithm of the width, in which latter case log T becomes incorporated with the first constant.

This property would make an excellent index of ductility, but the inherent difficulties of accurately determining with rapidity the average reduced dimensions of a plate specimen militate against its adoption in practice. Further, there is always the probability of latent flaws determining to some extent the final outline of the ruptured surfaces. In fact, any objections to the use of the reduction of area as the proper commercial measure of ductility apply here with equal force.

In Figs. 12, 13, and 14 the extensions given in Tables VI, VII, and VIII are again plotted, but in this instance, separately for each fixed length, to a base of $\frac{\text{width}}{\text{thickness}}$; that is, instead of indicating the variation of extension with change in length for different widths, these curves show the variation of extension with change in width for different lengths of bar. For each metal the curves, excluding that for the 1-inch length, are of the same type.

Dealing with Fig. 12, it may be said that the extension rises as the ratio is increased from 2 to about 7, remains almost constant between the

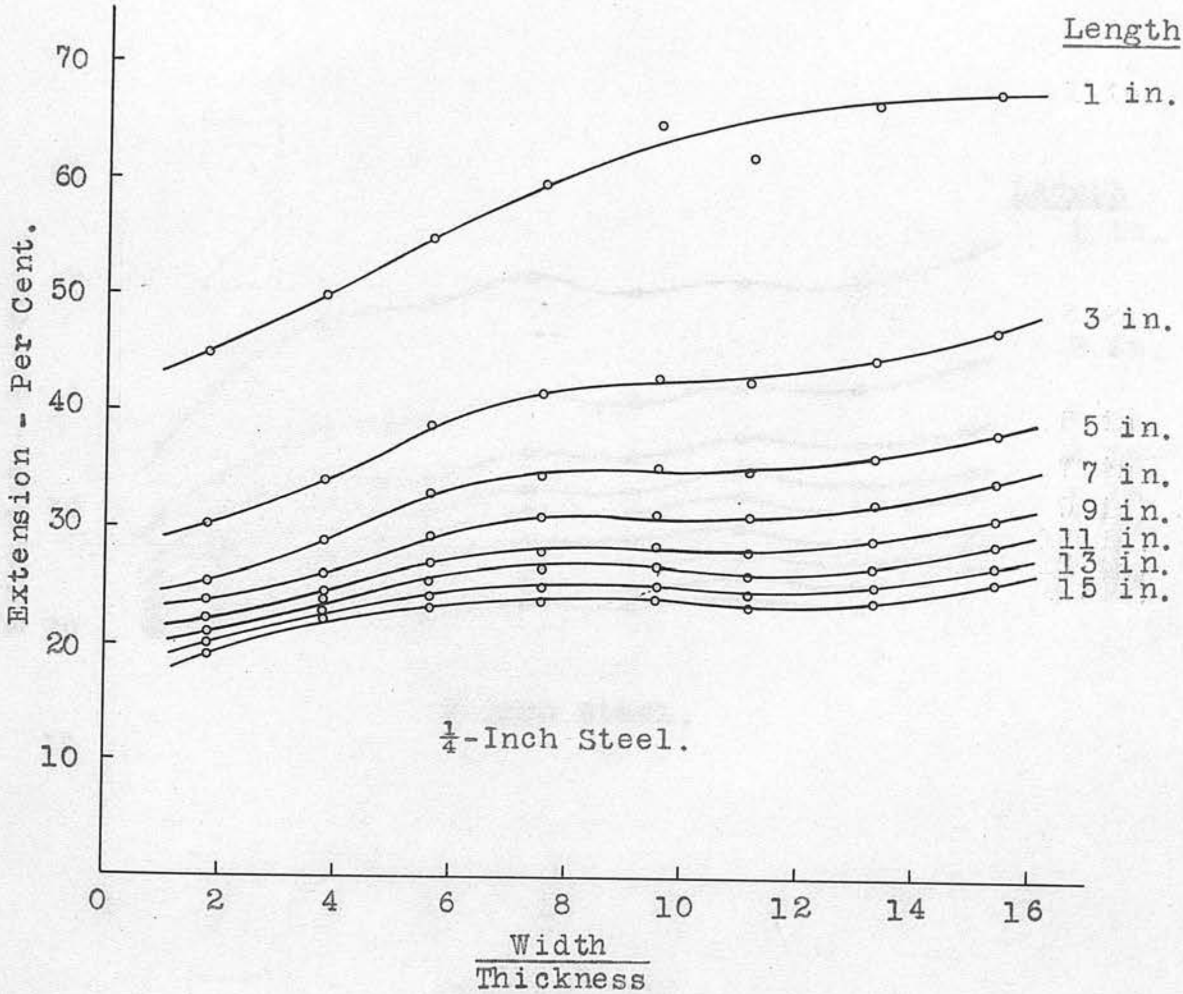

Length
1 in.
3 in.
5 in.
7 in.
9 in.
11 in.
13 in.
15 in.
Extension - Per Cent.
70
60
50
40
30
20
10
0
2
4
6
8
10
12
14
16
¼-Inch Steel.
Width / Thickness

Fig.12.

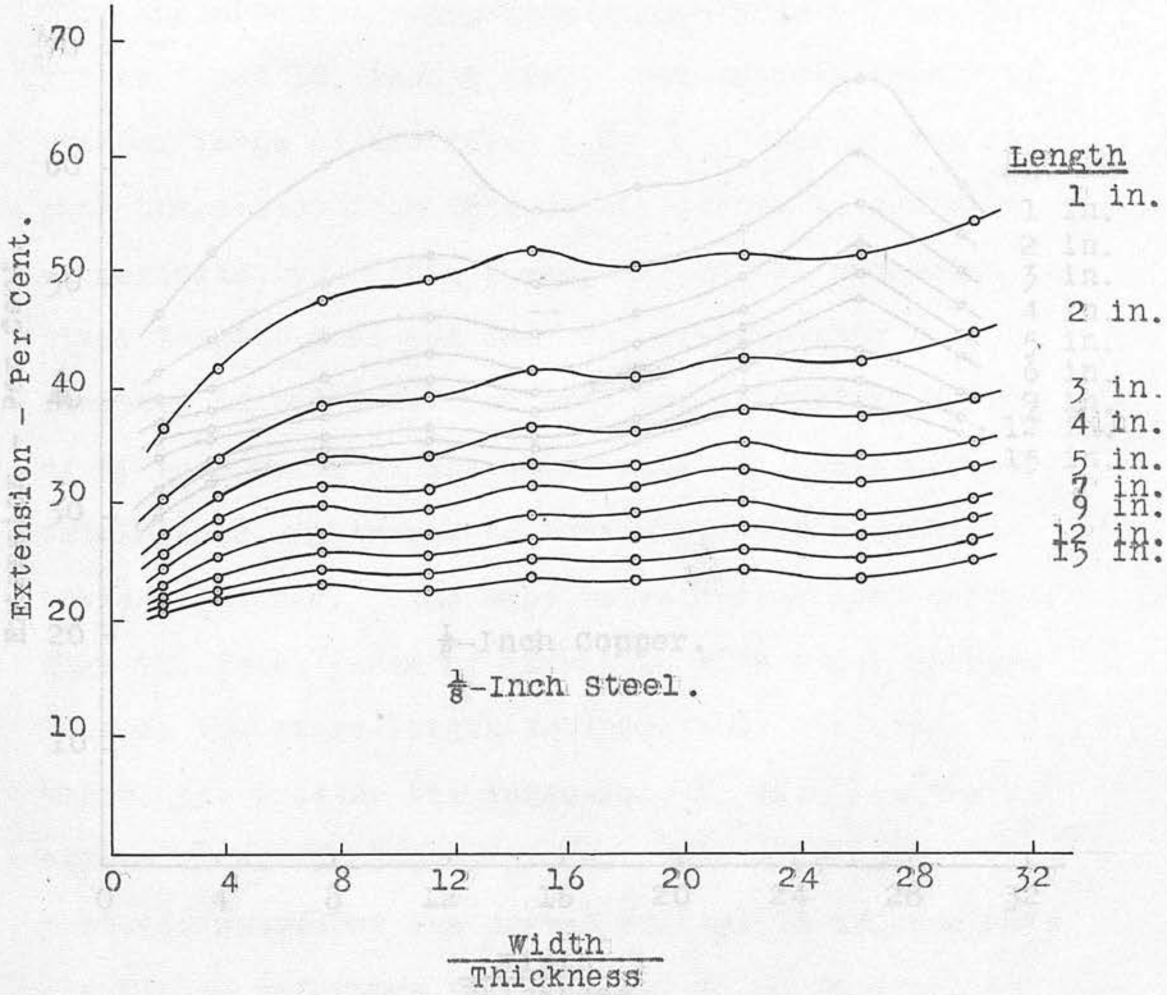

70
60
50
40
30
20
10
Extension - Per Cent.
0
4
8
12
16
20
24
28
32
Length
1 in.
2 in.
3 in.
4 in.
5 in.
7 in.
9 in.
12 in.
15 in.
$\frac{1}{8}$-Inch Steel.
Width / Thickness

Fig.13.

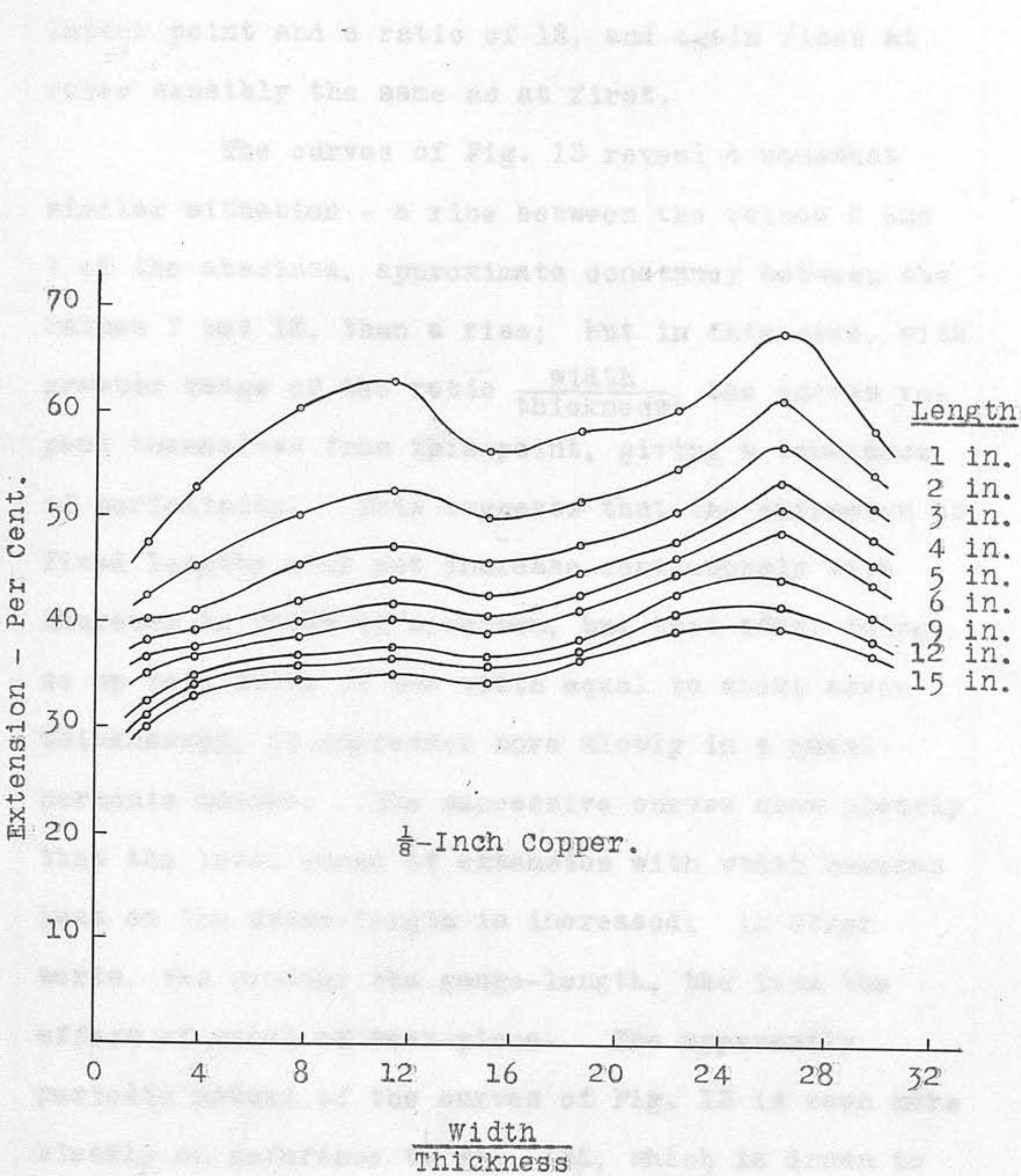

70
60
50
40
30
20
10
0
4
8
12
16
20
24
28
32
Extension - Per Cent.
Width / Thickness
$\frac{1}{8}$-Inch Copper.
Length
1 in.
2 in.
3 in.
4 in.
5 in.
6 in.
9 in.
12 in.
15 in.

Fig.14.

latter point and a ratio of 12, and again rises at rates sensibly the same as at first.

The curves of Fig. 13 reveal a somewhat similar situation - a rise between the values 2 and 7 of the abscissa, approximate constancy between the values 7 and 12, then a rise; but in this case, with greater range of the ratio $\frac{\text{width}}{\text{thickness}}$, the curves repeat themselves from this point, giving a semblance of periodicity. This suggests that the extension on fixed lengths does not increase continuously with increase in width of specimen, but that after doing so up to a value of the width equal to about seven thicknesses, it increases more slowly in a quasi-harmonic manner. The successive curves show clearly that the total range of extension with width becomes less as the gauge-length is increased; in other words, the greater the gauge-length, the less the effect of width of test-piece. The apparently periodic nature of the curves of Fig. 13 is seen more clearly on reference to Fig. 13A, which is drawn to larger scales. Maxima appear to occur at the values $7\frac{1}{2}$, 15, $22\frac{1}{2}$, and probably 30, of the abscissa, and minima about midway between them.

Fig. 14 shows similar curves for the copper. Here, after the initial or continuous rise,

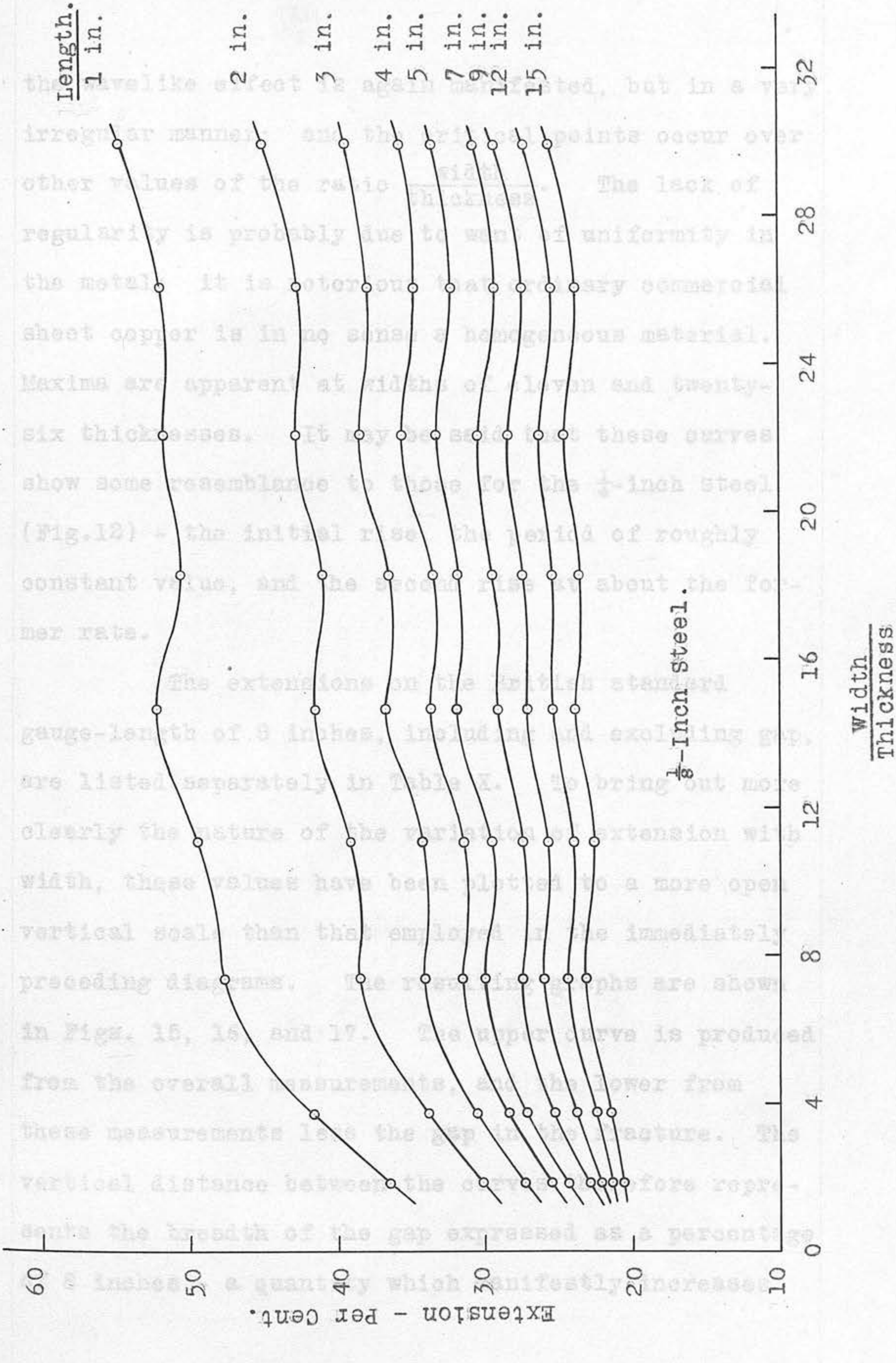

Length.
1 in.
2 in.
3 in.
4 in.
5 in.
7 in.
9 in.
12 in.
15 in.
1/8-Inch Steel.
Width / Thickness
Extension - Per Cent.
0
4
8
12
16
20
24
28
32
10
20
30
40
50
60

Fig. 13A.

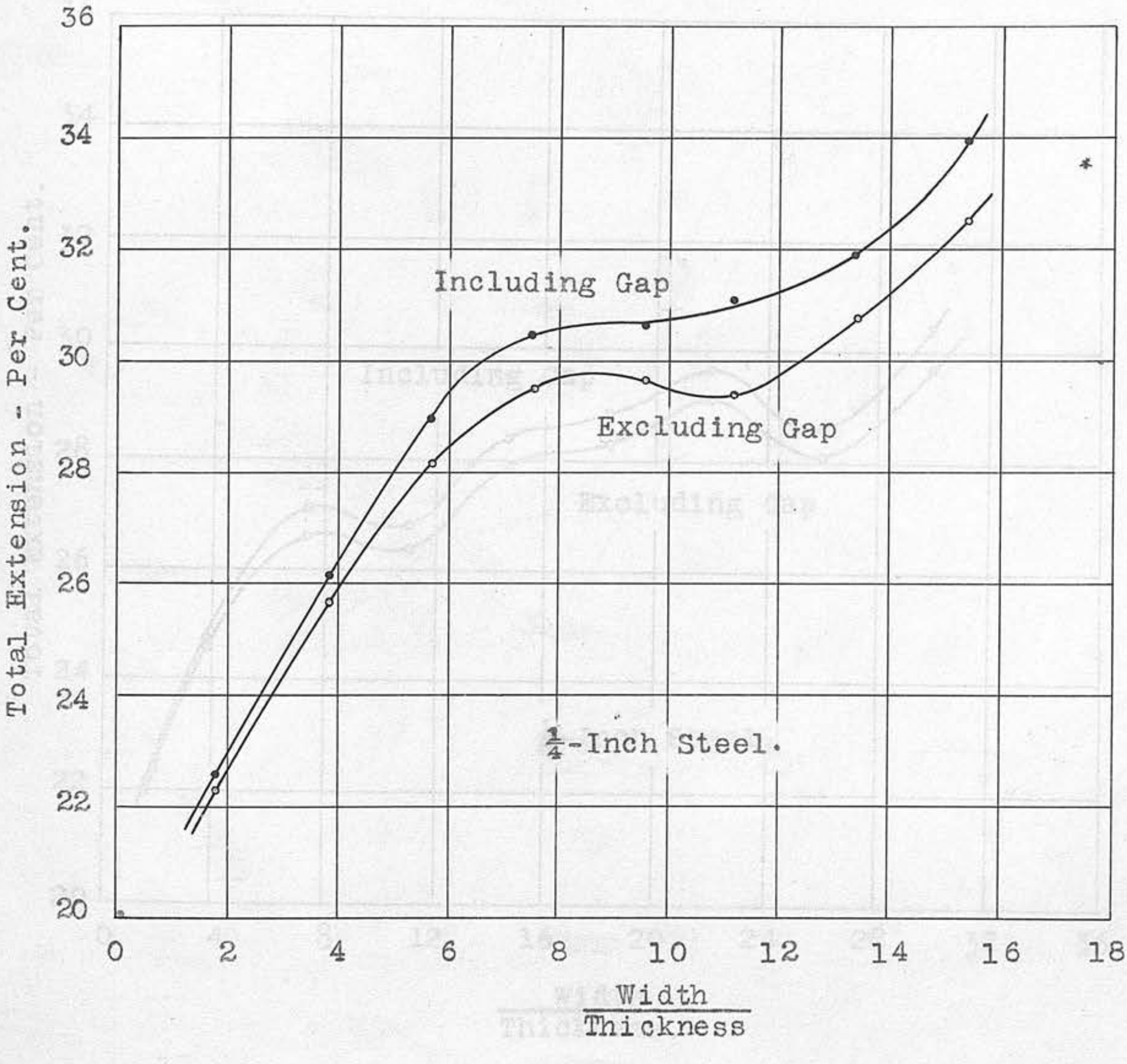
the wavelike effect is again manifested, but in a very irregular manner; and the critical points occur over other values of the ratio $\frac{\text{width}}{\text{thickness}}$. The lack of regularity is probably due to want of uniformity in the metal: it is notorious that ordinary commercial sheet copper is in no sense a homogeneous material. Maxima are apparent at widths of eleven and twenty-six thicknesses. It may be said that these curves show some resemblance to those for the $\frac{1}{4}$-inch steel (Fig.12) - the initial rise, the period of roughly constant value, and the second rise at about the former rate.

The extensions on the British standard gauge-length of 8 inches, including and excluding gap, are listed separately in Table X. To bring out more clearly the nature of the variation of extension with width, these values have been plotted to a more open vertical scale than that employed in the immediately preceding diagrams. The resulting graphs are shown in Figs. 15, 16, and 17. The upper curve is produced from the overall measurements, and the lower from these measurements less the gap in the fracture. The vertical distance between the curves therefore represents the breadth of the gap expressed as a percentage of 8 inches - a quantity which manifestly increases

TABLE X.

Extension on 8 Inches.

Nominal Width. Inches.	Extension per cent.					
	$\frac{1}{4}$-Inch Steel.		$\frac{1}{8}$-Inch Steel.		$\frac{1}{8}$-Inch Copper.	
	Including Gap.	Excluding Gap.	Including Gap.	Excluding Gap.	Including Gap.	Excluding Gap.
$\frac{1}{4}$	-	-	22.57	22.45	33.32	32.92
$\frac{1}{2}$	22.58	22.29	24.81	24.56	35.77	35.36
1	26.15	25.69	27.09	26.59	37.75	37.19
$1\frac{1}{2}$	28.98	28.15	26.82	26.45	38.86	38.27
2	30.44	29.48	28.44	27.94	38.87	37.22
$2\frac{1}{2}$	30.61	29.61	28.90	28.27	40.69	39.20
3	31.10	29.35	29.70	29.14	45.59	43.51
$3\frac{1}{2}$	31.90	30.71	28.61	28.11	47.27	45.72
4	33.93	32.47	30.50	29.75	44.07	41.57



Total Extension - Per Cent.
36
34
32
30
28
26
24
22
20
0
2
4
6
8
10
12
14
16
18
Including Gap
Excluding Gap
¼-Inch Steel.
Width / Thickness

Fig.15.

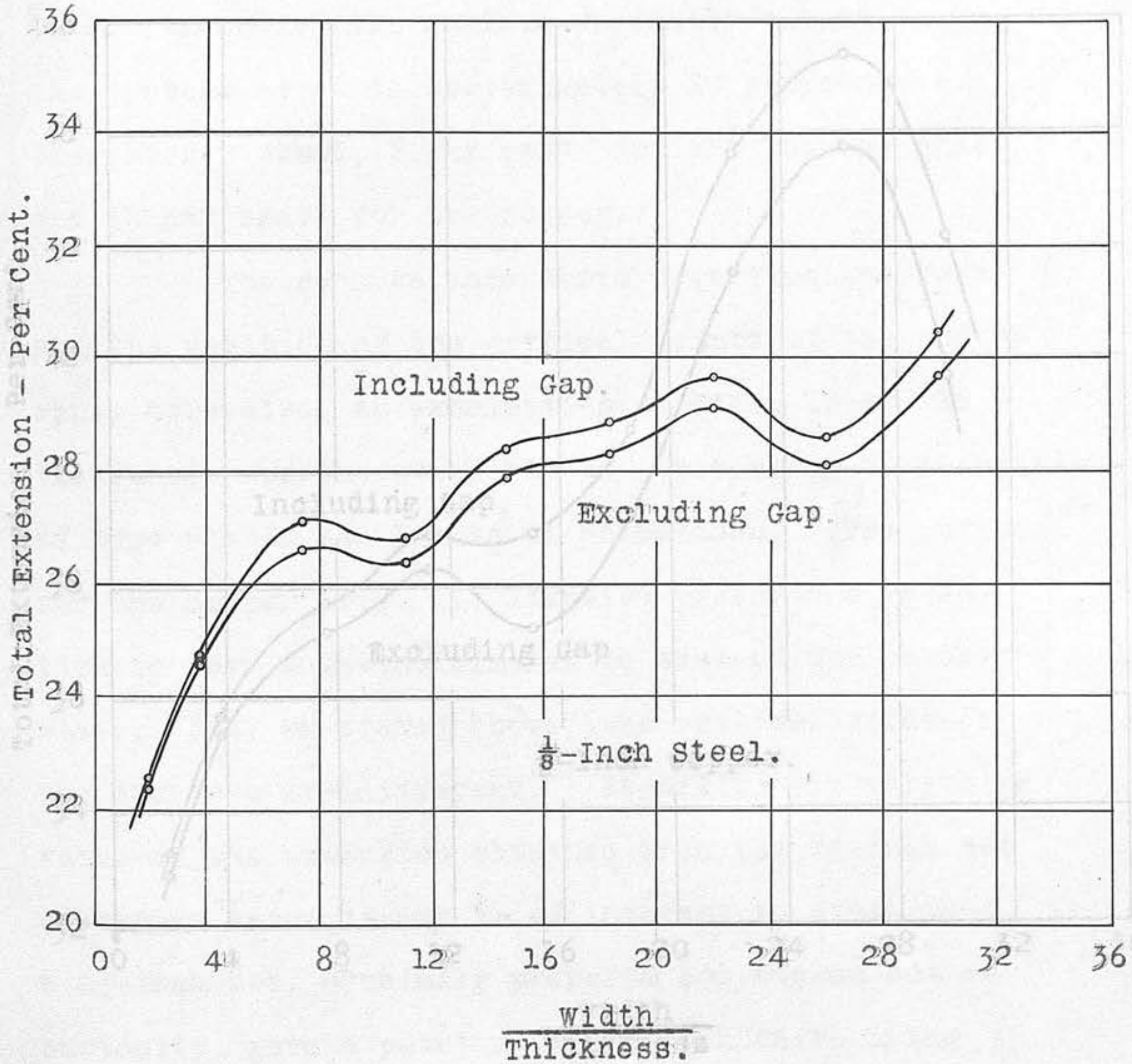

Total Extension - Per Cent.
36
34
32
30
28
26
24
22
20
0
4
8
12
16
20
24
28
32
36
Including Gap
Excluding Gap
$\frac{1}{8}$-Inch Steel.
Width / Thickness.

Fig.16.

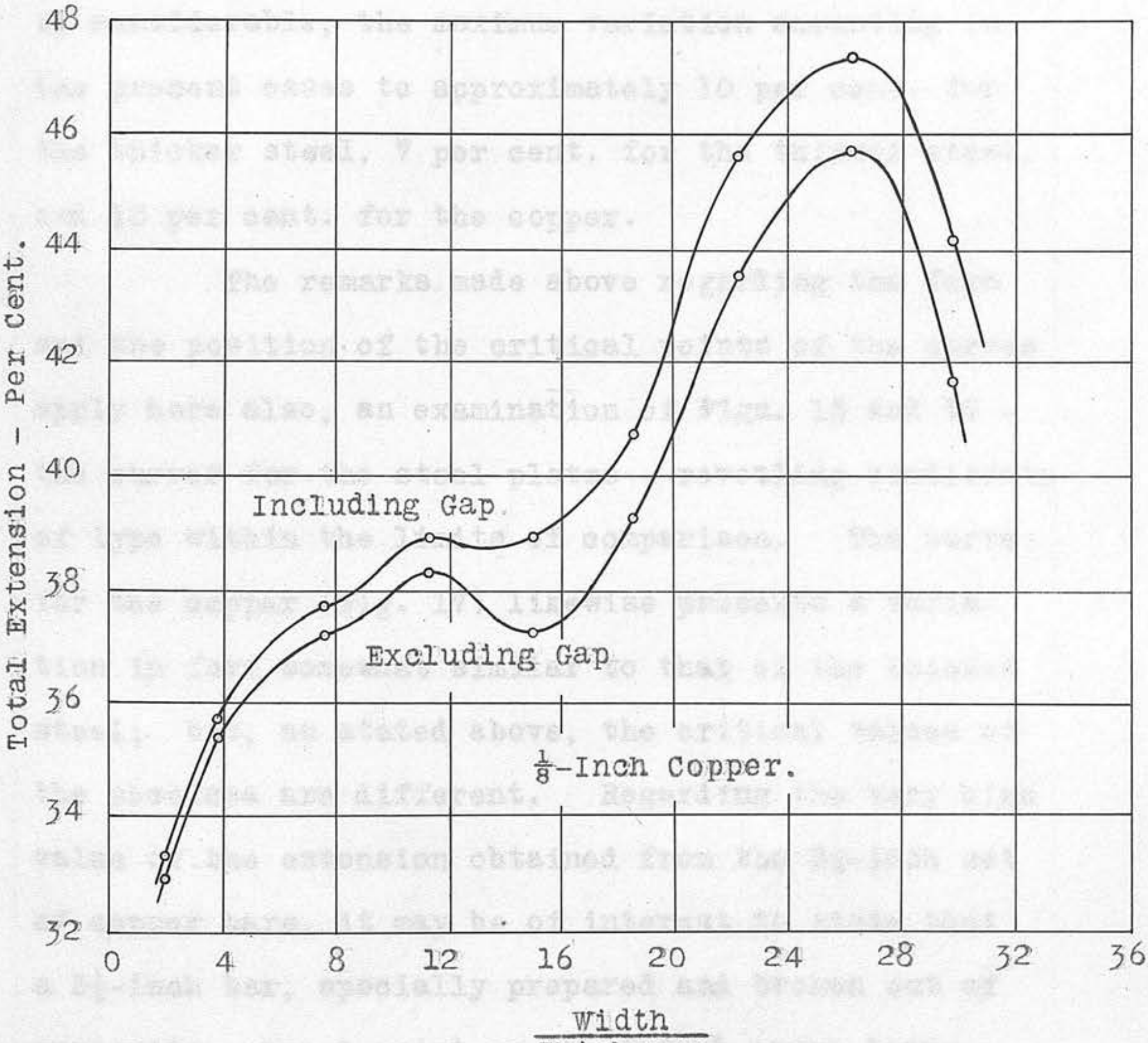

Total Extension - Per Cent.
48
46
44
42
40
38
36
34
32
0
4
8
12
16
20
24
28
32
36
Including Gap
Excluding Gap
$\frac{1}{8}$-Inch Copper.
Width / Thickness

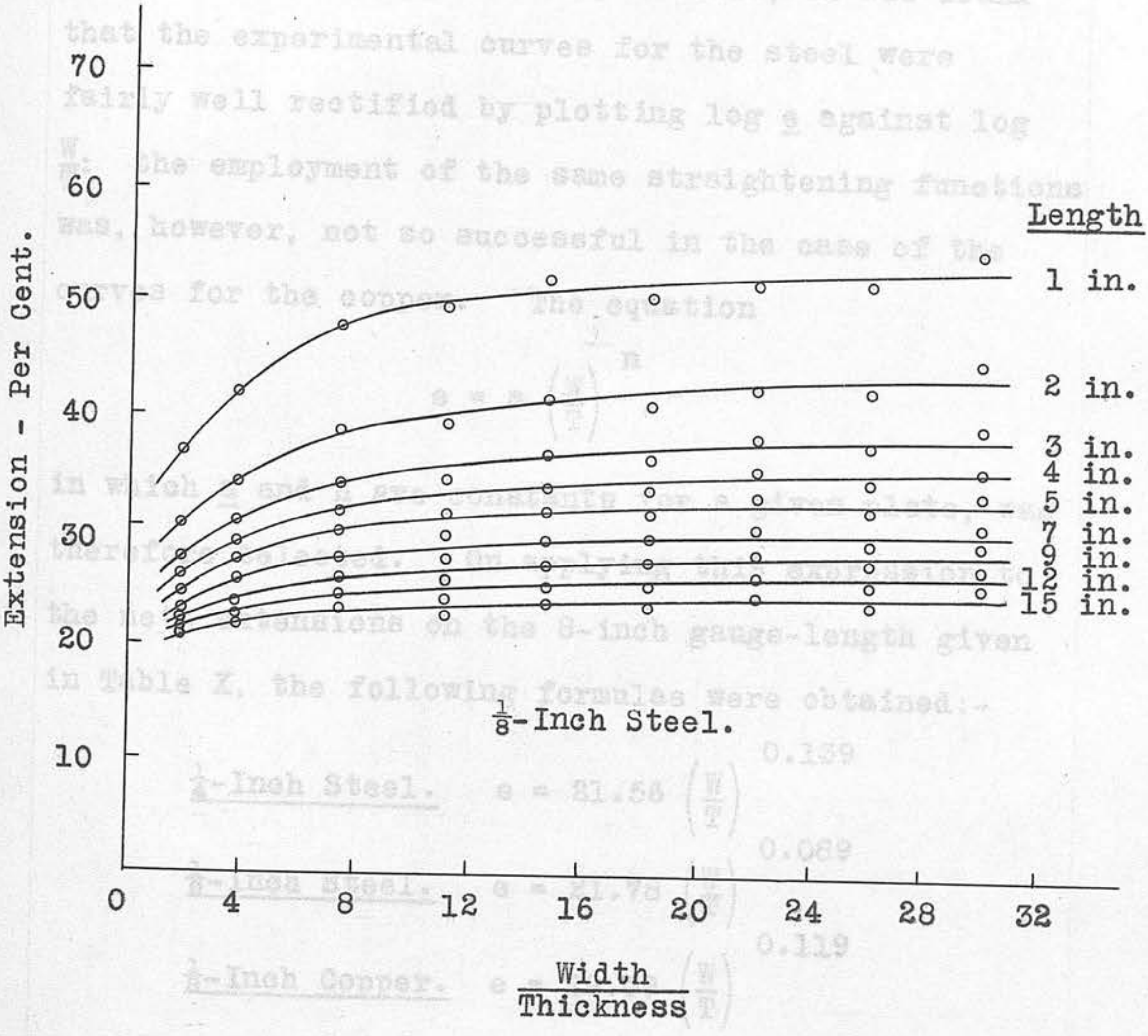
Fig. 17.

though somewhat irregularly, with the width of the specimen. The influence of the width of the test-bar on the percentage of elongation on this gauge-length is considerable, the maximum variation amounting in the present cases to approximately 10 per cent. for the thicker steel, 7 per cent. for the thinner steel, and 13 per cent. for the copper.

The remarks made above regarding the form and the position of the critical points of the curves apply here also, an examination of Figs. 15 and 16 - the curves for the steel plates - revealing similarity of type within the limits of comparison. The curve for the copper (Fig. 17) likewise presents a variation in form somewhat similar to that of the thicker steel; but, as stated above, the critical values of the abscissa are different. Regarding the very high value of the extension obtained from the $3\frac{1}{2}$-inch set of copper bars, it may be of interest to state that a $3\frac{1}{4}$-inch bar, specially prepared and broken out of curiosity, gave a point on the 8-inch curve lying about midway between those for the 3-inch and $3\frac{1}{2}$-inch sets.

All the extension-width curves for fixed lengths, of which those for the 8-inch gauge-length may be taken as typical, have thus certain features in common. Of these, the maximum at the end of the first and most regular portion is doubtless that discovered by Barba. The general trend in all cases is upwards.

Plotted to the closer vertical scale of Figs. 12, 13, and 14, the points might be regarded as lying on or about a smooth parabolic curve of the form $y = ax^n$. This is rendered more apparent on reference to Fig. 13B which again shows the points plotted from the values of the extension on fixed lengths of the thinner steel specimens, but in this case with a continuous or smooth parabolic curve drawn through their midst. If this assumption be made, the discussion becomes very simple. On the other hand, if the succeeding maxima and minima represent the real condition, a periodic element is introduced at a certain stage, and the matter, in consequence, becomes much more complicated. The only satisfactory method of deciding which of these views is correct would have been to examine the nature of the continuity of the curves in the neighbourhood of the apparent turning values by testing specimens cut from the same plates and differing in width by, say, $\frac{1}{8}$ inch. Unfortunately, the whole of the metal had been used up in the test-bars proper, and consequently this



Extension - Per Cent.
70
60
50
40
30
20
10
0
4
8
12
16
20
24
28
32
Length
1 in.
2 in.
3 in.
4 in.
5 in.
7 in.
9 in.
12 in.
15 in.
1/8-Inch Steel.
Width/Thickness

Fig.13B.

particular enquiry was postponed until the plates referred to later were available.

On the former assumption, namely, that the extension-width relation is parabolic, it was found that the experimental curves for the steel were fairly well rectified by plotting log e against log $\frac{W}{T}$: the employment of the same straightening functions was, however, not so successful in the case of the curves for the copper. The equation

$$e = a\left(\frac{W}{T}\right)^{n},$$

in which a and n are constants for a given plate, was therefore selected. On applying this expression to the nett extensions on the 8-inch gauge-length given in Table X, the following formulae were obtained:-

$\frac{1}{4}$-Inch Steel. $e = 21.56\left(\frac{W}{T}\right)^{0.139}$

$\frac{1}{8}$-Inch Steel. $e = 21.78\left(\frac{W}{T}\right)^{0.089}$

$\frac{1}{8}$-Inch Copper. $e = 29.09\left(\frac{W}{T}\right)^{0.119}$

The extensions calculated from these formulae are given in Table XA. In the case of the thicker steel the residuals, with the exception of the first and last entries, are quite satisfactory. Again the thinner steel shows its superiority in the

TABLE XA.

Calculated Values of Extension on 8 Inches.
(Parabolic)

Nominal Width. Inches.	Extension per cent.								
	$\frac{1}{4}$-Inch Steel.			$\frac{1}{8}$-Inch Steel.			$\frac{1}{8}$-Inch Copper.		
	Observed.	Calculated.	Difference.	Observed.	Calculated.	Difference.	Observed.	Calculated.	Difference.
$\frac{1}{4}$	-	-	-	22.5	22.9	-0.4	32.9	31.4	+1.5
$\frac{1}{2}$	22.3	23.3	-1.0	24.6	24.4	+0.2	35.4	34.1	+1.3
1	25.7	26.0	-0.3	26.6	26.0	+0.6	37.2	37.1	+0.1
$1\frac{1}{2}$	28.1	27.4	+0.7	26.5	26.9	-0.4	38.3	38.8	-0.5
2	29.5	28.6	+0.9	27.9	27.6	+0.3	37.2	40.2	-3.0
$2\frac{1}{2}$	29.6	29.5	+0.1	28.3	28.2	+0.1	39.2	41.2	-2.0
3	29.4	30.1	-0.7	29.1	28.6	+0.5	43.5	42.1	+1.4
$3\frac{1}{2}$	30.7	30.9	-0.2	28.1	29.0	-0.9	45.7	42.9	+2.8
4	32.5	31.5	+1.0	29.7	29.4	+0.3	41.6	43.6	-2.0

matter of uniformity of quality, the differences in this case being practically negligible. The copper, on the other hand, is most unsatisfactory; from the present point of view, the large discrepancies between the observed and the calculated values must be put down to variation in quality of the material. The simple measures of precision used above, namely, the arithmetic mean and the algebraic sum of the residuals, give 0.6 and +0.5 per cent. respectively for the $\frac{1}{4}$-inch steel, 0.4 and +0.3 per cent. for the $\frac{1}{8}$-inch steel, and 1.6 and -0.4 per cent. for the $\frac{1}{8}$-inch copper.

In the absence of further experimental data, the presumption is that, with high-grade ductile metal, the influence of the width of the specimen is such that the extension on a fixed length follows a law of the type $e = a\left(\frac{W}{T}\right)^n$, in which a and n are positive constants. It is of interest to note that, for the plates and gauge-length under consideration, the values of the constant a for the two thicknesses of steel are almost identical. The index n is fractional and lies in the neighbourhood of 0.1; further, it appears to differ slightly with the material and with the absolute thickness of the plate. As the thickness was practically uniform in each instance, the reciprocal of its <u>n</u>th power may be incorporated with the constant <u>a</u>, from which it would appear that, so far as these researches indicate, the extension varies approximately as the tenth root of the width.

Further study of the values of the extension on 8 inches revealed the fact that the extension-width relation for this fixed or constant gauge-length could be equally well represented by the simple logarithmic equation

$$e = a + b \log \frac{W}{T},$$

where <u>a</u> and <u>b</u> are positive constants. This doubtless follows from the flatness of the greater portion of the curves of extension on fixed lengths, as shown, for example, in Fig. 13B. The actual formulae obtained were

<u>$\frac{1}{4}$-Inch Steel.</u> $e = 21.01 + 8.780 \log \frac{W}{T}$

<u>$\frac{1}{8}$-Inch Steel.</u> $e = 21.38 + 5.415 \log \frac{W}{T}$

<u>$\frac{1}{8}$-Inch Copper.</u> $e = 27.61 + 10.81 \log \frac{W}{T}$.

As would be expected, the values of the first constant for both sets of steel specimens are again almost equal: further, they differ little numerically from the values of the first constant in the corresponding parabolic equations. On substituting in these empirical formulae the various values of the independent variable, the figures listed in Table XB were obtained. The arithmetic means of the residuals, without regard to sign, are 0.6, 0.4, and 1.7 per cent. for the thicker steel, the thinner steel, and the copper respectively; and the corresponding algebraic sums are +0.1, +0.1, and +0.2 per cent., results showing, if anything, a slight improvement on those recorded for the parabolic residuals. On this basis, the percentage of elongation on the 8-inch gauge-length or, more generally, on any fixed datum-length of test-bars cut from one and the same plate may therefore be regarded as a linear function of the logarithm of the width of the specimen.

A third attempt to fit an equation to the curve for the 8-inch gauge-length was confined to the extensions obtained from the thinner steel plate. The exponential relation

$$e = a\epsilon^{b\frac{W}{T}} + c$$

was tried, and yielded results almost as good as those derived from the parabolic and logarithmic

TABLE XB.

Calculated Values of Extension on 8 Inches.
(Logarithmic)

Nominal Width. Inches.	Extension per cent.								
	$\frac{1}{4}$-Inch Steel.			$\frac{1}{8}$-Inch Steel.			$\frac{1}{8}$-Inch Copper.		
	Observed.	Calculated.	Difference.	Observed.	Calculated.	Difference	Observed.	Calculated.	Difference.
$\frac{1}{4}$	-	-	-	22.5	22.8	-0.3	32.9	30.6	+2.3
$\frac{1}{2}$	22.3	23.2	-0.9	24.6	24.4	+0.2	35.4	33.8	+1.6
1	25.7	26.1	-0.4	26.6	26.1	+0.5	37.2	37.2	$\pm$0.0
$1\frac{1}{2}$	28.1	27.6	+0.5	26.5	27.0	-0.5	38.3	39.0	-0.7
2	29.5	28.7	+0.8	27.9	27.7	+0.2	37.2	40.3	-3.1
$2\frac{1}{2}$	29.6	29.6	$\pm$0.0	28.3	28.2	+0.1	39.2	41.3	-2.1
3	29.4	30.2	-0.8	29.1	28.6	+0.5	43.5	42.2	+1.3
$3\frac{1}{2}$	30.7	30.9	-0.2	28.1	29.0	-0.9	45.7	42.9	+2.8
4	32.5	31.4	+1.1	29.7	29.4	+0.3	41.6	43.5	-1.9

forms. In this equation the constants a and b are negative; c, the asymptote, is positive; and ϵ, as before, is the Napierian base. 'Rectification' of the curve on this basis yielded the formula

$$e = 30 - 8.123\epsilon^{-0.0912\frac{W}{T}},$$

from which the calculated values listed in Table XC have been determined. Application to the residuals of the simple tests used above gives 0.4 per cent. for the mean value and -0.8 per cent. for the algebraic sum. As regards 'fit' there is little to choose between the three types of relation employed, but on the score of simplicity preference would naturally be given to the first or second.

On the assumption that a periodic element is present in the extension-width relation, it becomes necessary to attempt some explanation of the phenomenon. It would, however, be absurd to derive formulae to fit the curves, the data being insufficient. In the joint papers [(1) (2)] referred to in Art. 2, the writer first with Dr Gulliver and later with Professor Hudson Beare advanced a tentative theory to account for the peculiar form of the extension-width curve for fixed lengths. It may be

TABLE XC.

$\frac{1}{8}$-Inch Steel - Calculated Values of Extension on 8 Inches.
(Exponential)

Nominal Width.	Extension per cent.		
Inches.	Observed.	Calculated.	Difference.
$\frac{1}{4}$	22.5	23.1	-0.6
$\frac{1}{2}$	24.6	24.2	+0.4
1	26.6	25.8	+0.8
$1\frac{1}{2}$	26.5	27.0	-0.5
2	27.9	27.9	±0.0
$2\frac{1}{2}$	28.3	28.5	-0.2
3	29.1	28.9	+0.2
$3\frac{1}{2}$	28.1	29.2	-1.1
4	29.7	29.5	+0.2

stated briefly as follows. 'If the type of contraction were geometrically constant, the extension would increase continuously with the width, the gauge-length remaining fixed. The fact that the extension does not do so suggests that there is some change in the process of deformation whereby less contraction, and therefore less extension, is produced in a wide bar than in a narrow one. On the other hand, the extension on a fixed length of a wide bar is greater than that of a narrow bar because the length is relatively less. These two contrary factors would influence the magnitude of the extension in opposite directions, and, within certain limits of the ratio of width to thickness, which probably occur in regular cycles, might approximately neutralise each other. Outside of these limits, shorter relative length is the determining factor, and the values of the extension tend upwards.' It has been pointed out earlier in this Article that the greater the width of the specimen, the less is the extension in the fracture. In later Articles it will be shown that the proportional reduction in width, and consequently the extension in the contracted region, is actually less the wider the bar.

From results such as the above, it is

impossible to generalise: further enquiry became necessary. This was carried out later on $\frac{1}{4}$-inch, $\frac{3}{8}$-inch, and $\frac{1}{2}$-inch mild steel boiler plates, all of which were rolled from slabs cut from a single ingot of acid Siemens steel, and annealed together.

Generally, the extensions on fixed lengths plotted much more smoothly than even those for the $\frac{1}{8}$-inch steel of the present investigations, and showed not the slightest indication of definite maxima. The curves of Fig. 13B may accordingly be taken as representing the extension-width relation for constant gauge-lengths in the case of very high-grade steel. It will be found that, for the most part, empirical formulae have been based on the values obtained from the specimens of the $\frac{1}{8}$-inch or Dalzell plate.

To avoid unnecessary repetition and useless bulk, the results of this merely confirmatory work have not been included in the Thesis.

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7. Extension on Lengths Proportional to $\sqrt{\text{Area}}$.

The Law of Similarity, stated in Art. 1, was first formulated for circular test-bars by Lebasteur and Marié (16), afterwards extended to the case of rectangular specimens by Barba (17), and later still made more general by Kick (18) in his Law of Proportional Resistances. Regarded from the most general point of view, this law might readily be taken to be true a priori, at least there appears to be no cogent reason to expect any other result; and, within the unavoidable differences of material and the ordinary limits of error in testing, experiment has repeatedly demonstrated its validity. It follows, therefore, that specimens of geometrically similar form give equal relative elongations and absolute elongations proportional to the dimensions of the test pieces; in other words, the extensions obtained from truly proportional bars are strictly comparable. Of course, it is to be understood that all secondary conditions, such as manner of gripping, rate of loading, etc., must also conform to the law. In ordinary commercial metal, however, lack of homogeneity is not always negligible, and frequently renders the verification of the law a matter of considerable difficulty.

Within limits, this law may be extended to the comparison of elongations obtained from bars of compact section although not similar from the geometrical point of view. Bauschinger (19) was the first to point out that, within the limits used in general practice in his day, the transverse dimensions and shape of section had no material effect on the extension, provided the gauge-length was made proportional to the square root of the cross-sectional area of the test-bar. This type of gauge-length is now universally employed on the Continent; and, since the War, its adoption in this country has met with some favour, especially in the testing of cold-drawn rods. For round, polygonal, and square bars, and for rectangular specimens of width not greater than about four or five thicknesses, it gives extensions that are readily comparable.

In this Article an attempt is made to determine the nature of the variation of extension on lengths proportional to the square root of the area of the specimens under consideration - flats of practically constant thickness and of varying width, giving a wide range of the ratio $\frac{\text{width}}{\text{thickness}}$.

The figures given in Tables XI, XII, and XIII were obtained by reading off from the large-

TABLE XI.

$\frac{1}{4}$-Inch Steel - Extension on Lengths Proportional to $\sqrt{Area}$.

Nominal Width. Inches.	Extension per cent. on a Length in Inches equal to				
	$\sqrt{Area}$.	$2\sqrt{Area}$.	$5\sqrt{Area}$.	$10\sqrt{Area}$.	$15\sqrt{Area}$.
$\frac{1}{2}$	59.0	50.0	37.7	28.7	25.3
1	59.2	50.0	36.6	28.8	25.7
$1\frac{1}{2}$	61.6	52.2	38.7	30.7	26.8
2	65.0	53.8	39.3	30.8	26.8
$2\frac{1}{2}$	68.8	55.7	38.4	29.8	26.0
3	64.7	52.2	36.8	28.6	24.6
$3\frac{1}{2}$	67.8	54.0	37.2	28.6	24.3
4	67.6	55.0	38.4	29.8	25.6

TABLE XII.

$\frac{1}{8}$-Inch Steel - Extension on Lengths Proportional to $\sqrt{\text{Area}}$.

Nominal Width.	Extension per cent. on a Length in Inches equal to				
Inches.	2 $\sqrt{\text{Area}}$.	5 $\sqrt{\text{Area}}$.	10 $\sqrt{\text{Area}}$.	15 $\sqrt{\text{Area}}$.	20 $\sqrt{\text{Area}}$.
$\frac{1}{4}$	50.0	37.6	30.8	27.9	26.1
$\frac{1}{2}$	52.4	38.7	31.7	28.6	27.0
1	51.7	39.6	32.2	29.1	27.1
$1\frac{1}{2}$	51.1	37.6	30.4	27.6	25.7*
2	51.3	38.2	31.4	28.1	26.0
$2\frac{1}{2}$	48.3	36.7	30.5	27.5	25.4
3	48.5	37.7	30.9	27.8	25.7
$3\frac{1}{2}$	47.5	36.2	29.3	26.3	24.3
4	49.2	36.7	30.3	27.7	25.7

TABLE XIII.

$\frac{1}{8}$-Inch Copper - Extension on Lengths Proportional to $\sqrt{\text{Area}}$.

Nominal Width.	Extension per cent. on a Length in Inches equal to				
Inches.	$2\sqrt{\text{Area}}$.	$5\sqrt{\text{Area}}$.	$10\sqrt{\text{Area}}$.	$15\sqrt{\text{Area}}$.	$20\sqrt{\text{Area}}$.
$\frac{1}{4}$	56.0	48.1	43.1	40.8	38.9
$\frac{1}{2}$	64.0	50.2	42.5	39.5	37.6
1	65.4	51.8	43.2	39.4	37.5
$1\frac{1}{2}$	65.2	51.5	43.0	39.5	37.8
2	56.2	47.5	40.4	37.4	36.5
$2\frac{1}{2}$	57.3	48.1	41.6	38.6	37.6
3	58.7	50.2	44.5	42.9	40.8
$3\frac{1}{2}$	65.2	55.8	47.5	43.4	40.5
4	56.7	49.1	42.3	39.3	37.2

scale extension-length graphs, referred to in the preceding Article, the extensions on lengths in inches equal to $\sqrt{\text{area}}$, $2\sqrt{\text{area}}$, etc. The curves of Figs. 18, 19, and 20 have been drawn from these figures, and indicate the influence of width of test-piece on this type of gauge-length. As regards comparability, the results show a decided improvement on those for fixed lengths, the figures for the larger values of $K\sqrt{A}$ manifesting some approach to constancy. Nevertheless, as only one transverse dimension is altered, there are still variations.

Regarding Fig. 18 in the light of the theory advanced in the previous Article to account for the quasi-periodic nature of the extension-width relation, it may be said that the change in the form of the contracted region appears to have a much greater effect in this case than in that of the constant gauge-length, with the result that the difference between the extreme values of the extension is considerably reduced. The curves from $2\sqrt{\text{area}}$ to $15\sqrt{\text{area}}$ are all of wave form and differ little, the extension increasing, decreasing, and again increasing as the width is increased continuously.

With greater range of the ratio $\frac{\text{width}}{\text{thickness}}$, the curves of Fig. 19 show the wave form of the

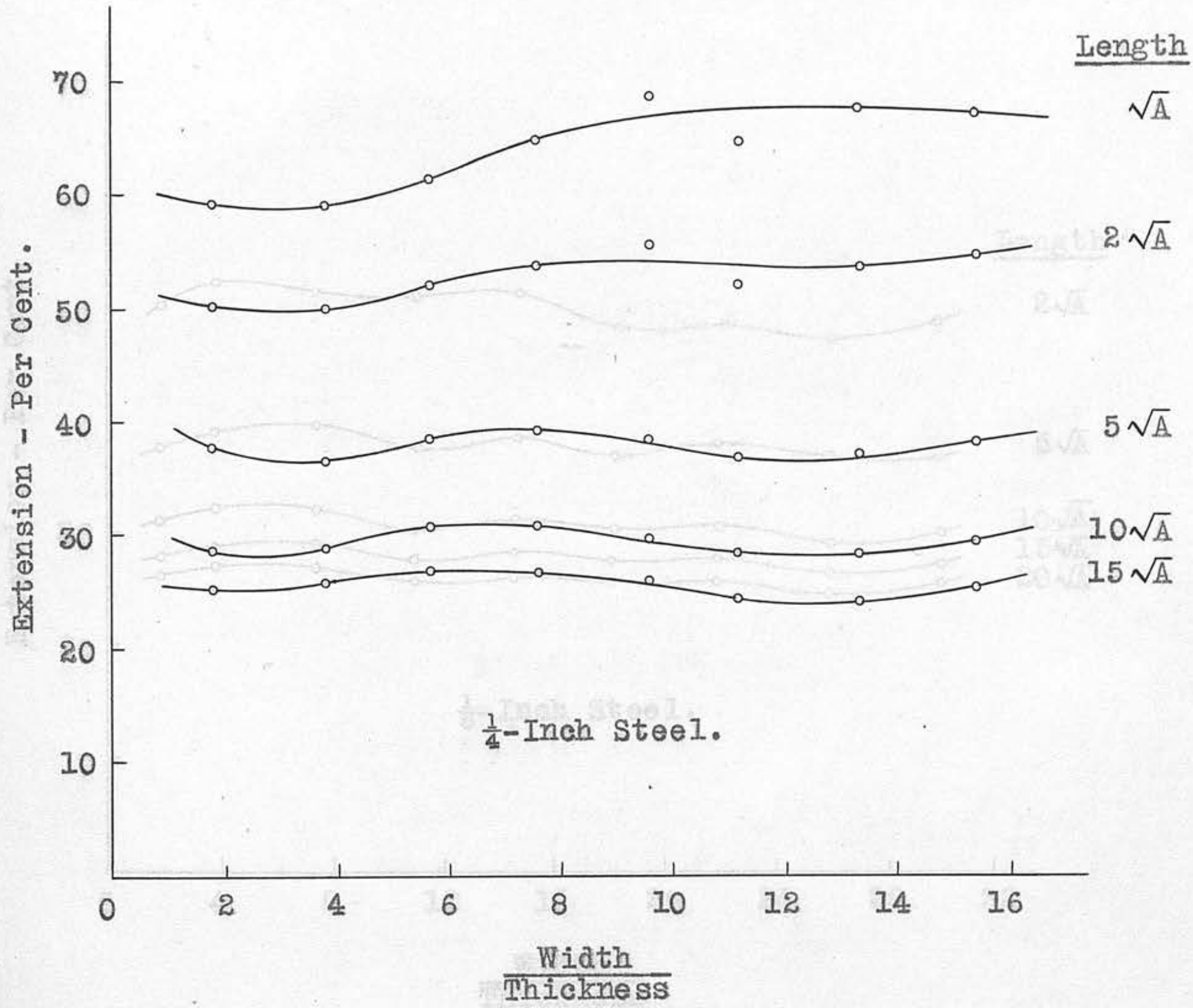

Length
√A
2 √A
5 √A
10√A
15 √A
70
60
50
40
30
20
10
Extension - Per Cent.
0
2
4
6
8
10
12
14
16
¼-Inch Steel.
Width / Thickness

Fig.18.

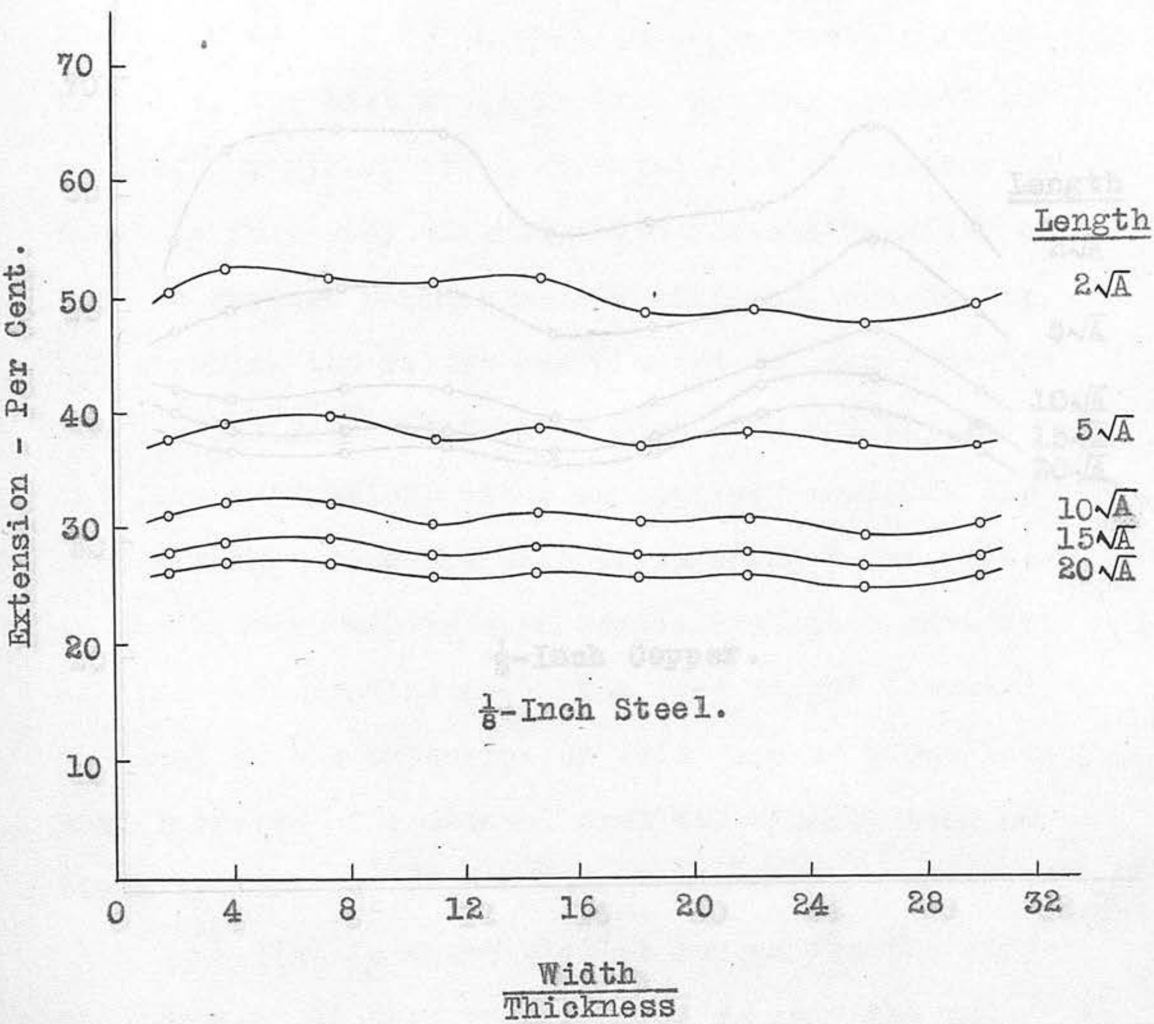

Extension - Per Cent.
70
60
50
40
30
20
10
0
4
8
12
16
20
24
28
32
Length
2√A
5√A
10√A
15√A
20√A
1/8-Inch Steel.
Width / Thickness

Fig.19.

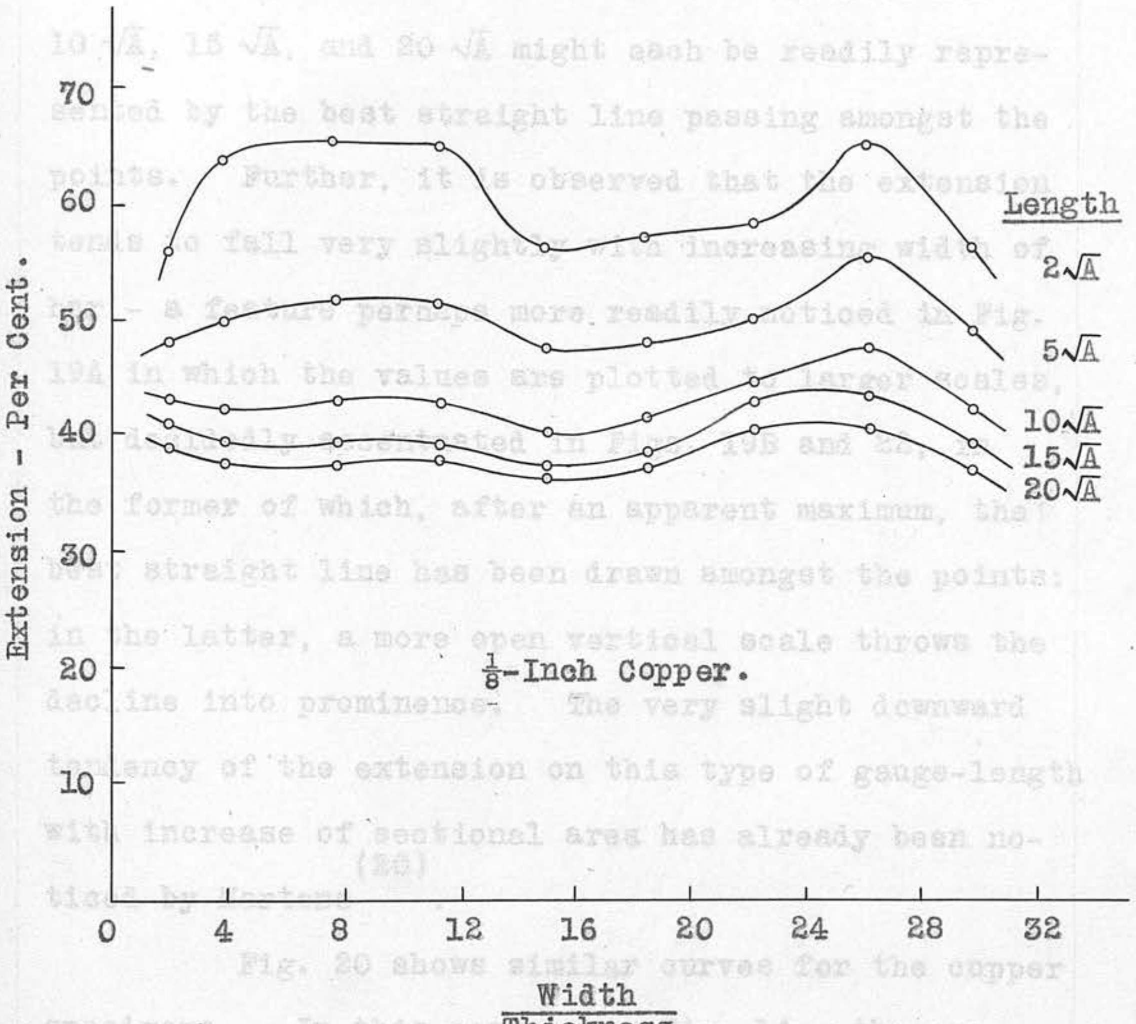

Extension - Per Cent.
70
60
50
40
30
20
10
0
4
8
12
16
20
24
28
32
Length
2√A
5√A
10√A
15√A
20√A
$\frac{1}{8}$-Inch Copper.
Width
Thickness

Fig.20.

preceding diagram repeating itself. Again it is to be noted that with increase of gauge-length, the influence of width of specimen on the extension becomes less marked; so much so, that the extensions for $10\sqrt{A}$, $15\sqrt{A}$, and $20\sqrt{A}$ might each be readily represented by the best straight line passing amongst the points. Further, it is observed that the extension tends to fall very slightly with increasing width of bar - a feature perhaps more readily noticed in Fig. 19A in which the values are plotted to larger scales, but decidedly accentuated in Figs. 19B and 22, in the former of which, after an apparent maximum, the best straight line has been drawn amongst the points: in the latter, a more open vertical scale throws the decline into prominence. The very slight downward tendency of the extension on this type of gauge-length with increase of sectional area has already been noticed by Martens[20].

Fig. 20 shows similar curves for the copper specimens. In this case as in Fig. 14 - the analogous diagram for the 8-inch gauge-length - the wave form is decidedly irregular, the extensions apparently varying in a most erratic manner. This lack of regularity is most probably due, as indicated above, to lack of uniformity in the metal. Very little com-

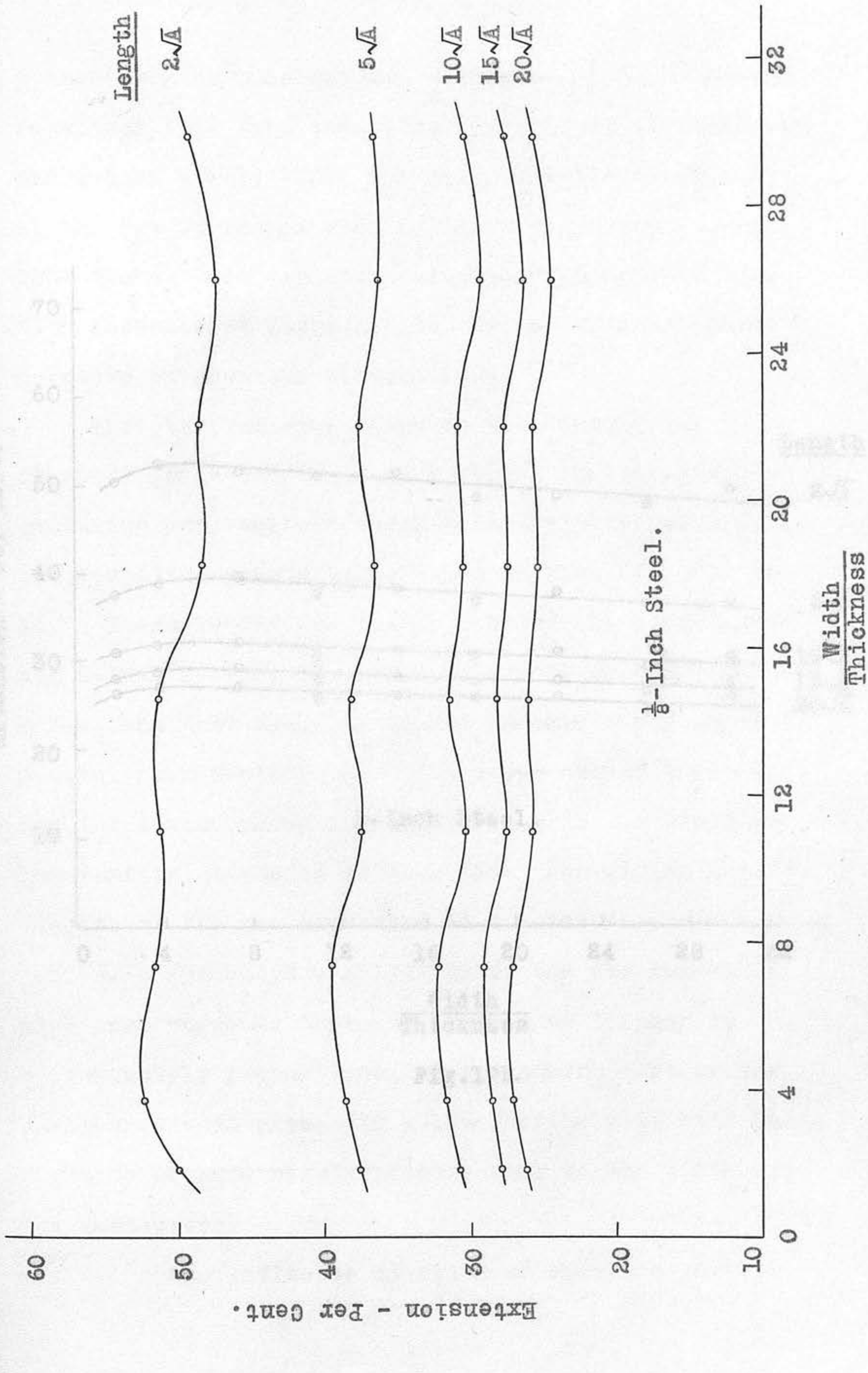

Length
2√A
5√A
10√A
15√A
20√A
1/8-Inch Steel.
Width / Thickness
Extension - Per Cent.
0
4
8
12
16
20
24
28
32
10
20
30
40
50
60

Fig.19A.

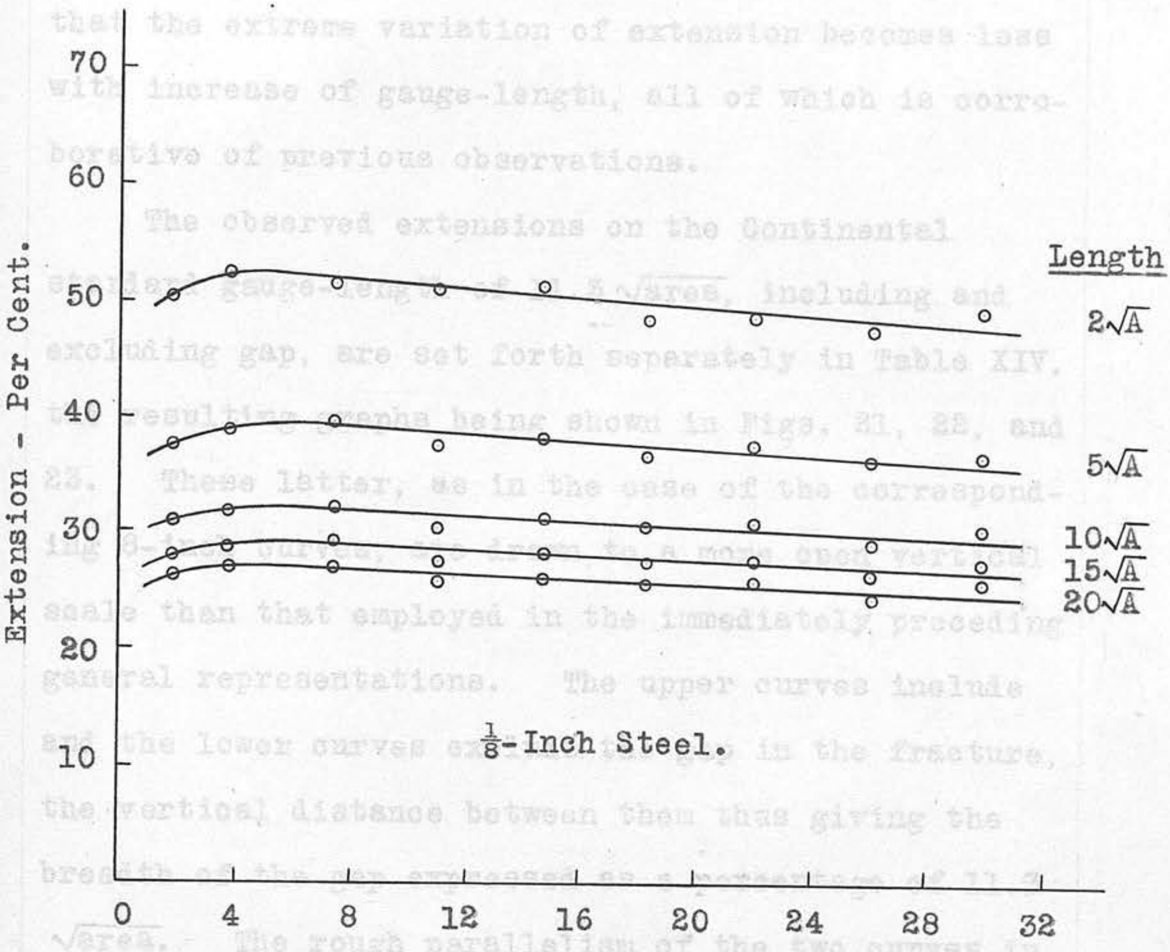

70
60
50
40
30
20
10
Extension - Per Cent.
Length
$2\sqrt{A}$
$5\sqrt{A}$
$10\sqrt{A}$
$15\sqrt{A}$
$20\sqrt{A}$
$\frac{1}{8}$-Inch Steel.
0
4
8
12
16
20
24
28
32
$\frac{\text{Width}}{\text{Thickness}}$

Fig.19B.

comment may be made on such curves. It may, however, be stated that they show some resemblance to those of the $\frac{1}{4}$-inch steel; that the singularly large extension of the set $3\frac{1}{2}$ inches wide is again manifested; and that the extreme variation of extension becomes less with increase of gauge-length, all of which is corroborative of previous observations.

The observed extensions on the Continental standard gauge-length of $11.3\sqrt{\text{area}}$, including and excluding gap, are set forth separately in Table XIV, the resulting graphs being shown in Figs. 21, 22, and 23. These latter, as in the case of the corresponding 8-inch curves, are drawn to a more open vertical scale than that employed in the immediately preceding general representations. The upper curves include and the lower curves exclude the gap in the fracture, the vertical distance between them thus giving the breadth of the gap expressed as a percentage of $11.3\sqrt{\text{area}}$. The rough parallelism of the two curves in each case suggests that the breadth of the gap is approximately proportional to the square root of the cross-sectional area: in a later Article it will be shown to be more nearly proportional to the width of the test-piece.

The influence of width of specimen on this

TABLE XIV.

Extension on $11.3\sqrt{\text{Area}}$.

Nominal Width. Inches.	Extension per cent.					
	$\frac{1}{4}$-Inch Steel.		$\frac{1}{8}$-Inch Steel.		$\frac{1}{8}$-Inch Copper.	
	Including Gap.	Excluding Gap.	Including Gap.	Excluding Gap.	Including Gap.	Excluding Gap.
$\frac{1}{4}$	-	-	30.33	29.85	43.89	42.34
$\frac{1}{2}$	28.18	27.57	31.42	30.73	42.46	41.30
1	28.79	28.13	32.10	31.14	43.01	41.90
$1\frac{1}{2}$	30.21	29.24	29.99	29.40	42.78	41.84
2	30.44	29.48	31.01	30.33	41.56	39.30
$2\frac{1}{2}$	29.39	28.51	30.46	29.70	42.46	40.64
3	28.75	27.32	30.56	29.94	46.22	43.90
$3\frac{1}{2}$	28.17	27.28	28.86	28.35	47.74	46.13
4	29.70	28.68	30.28	29.56	43.74	41.32

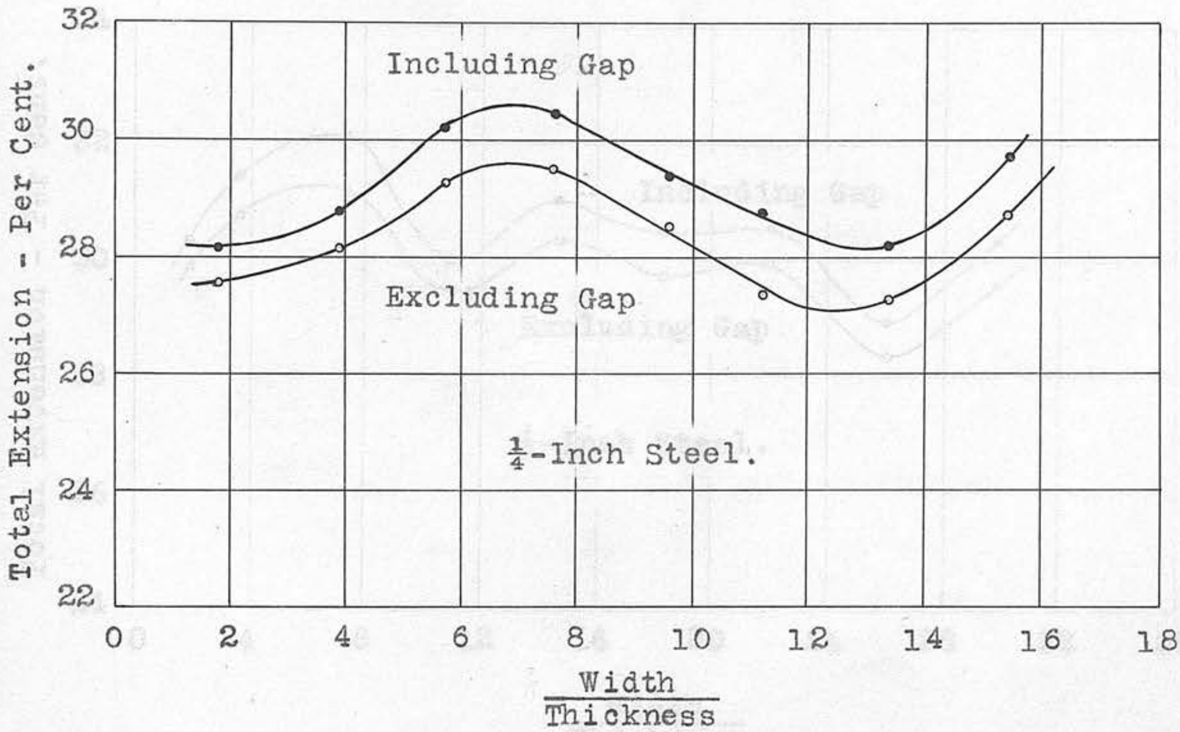
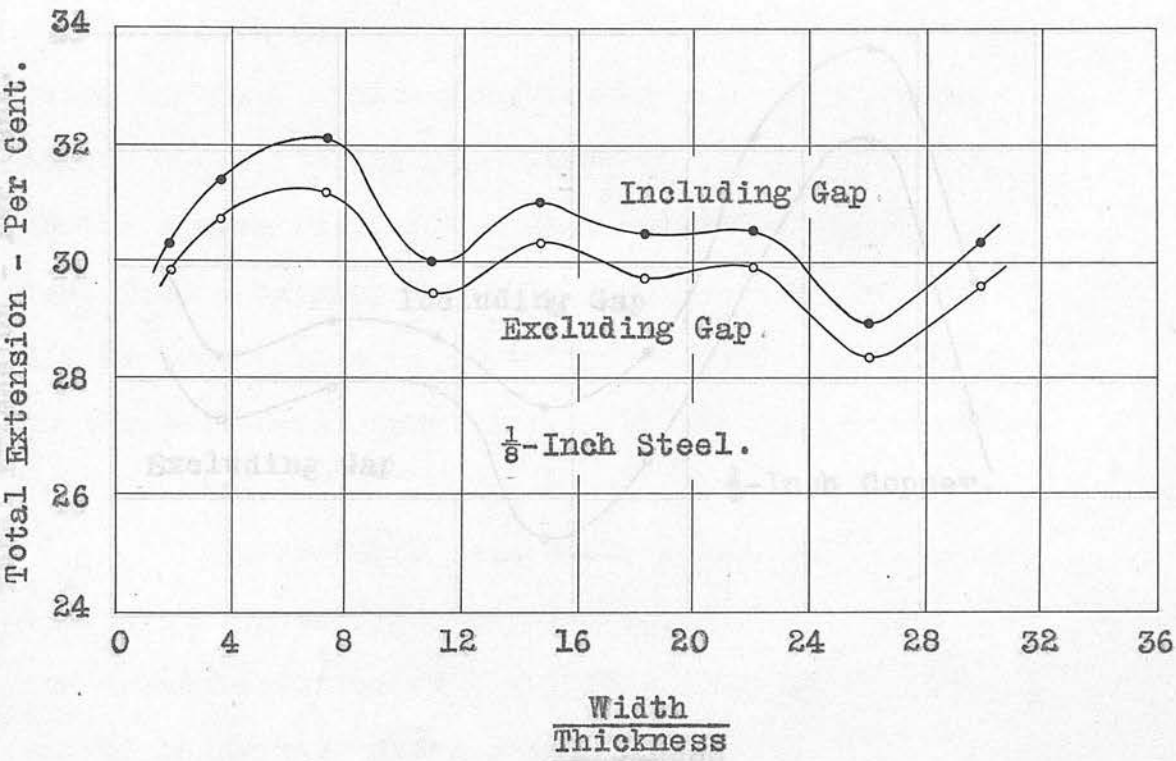
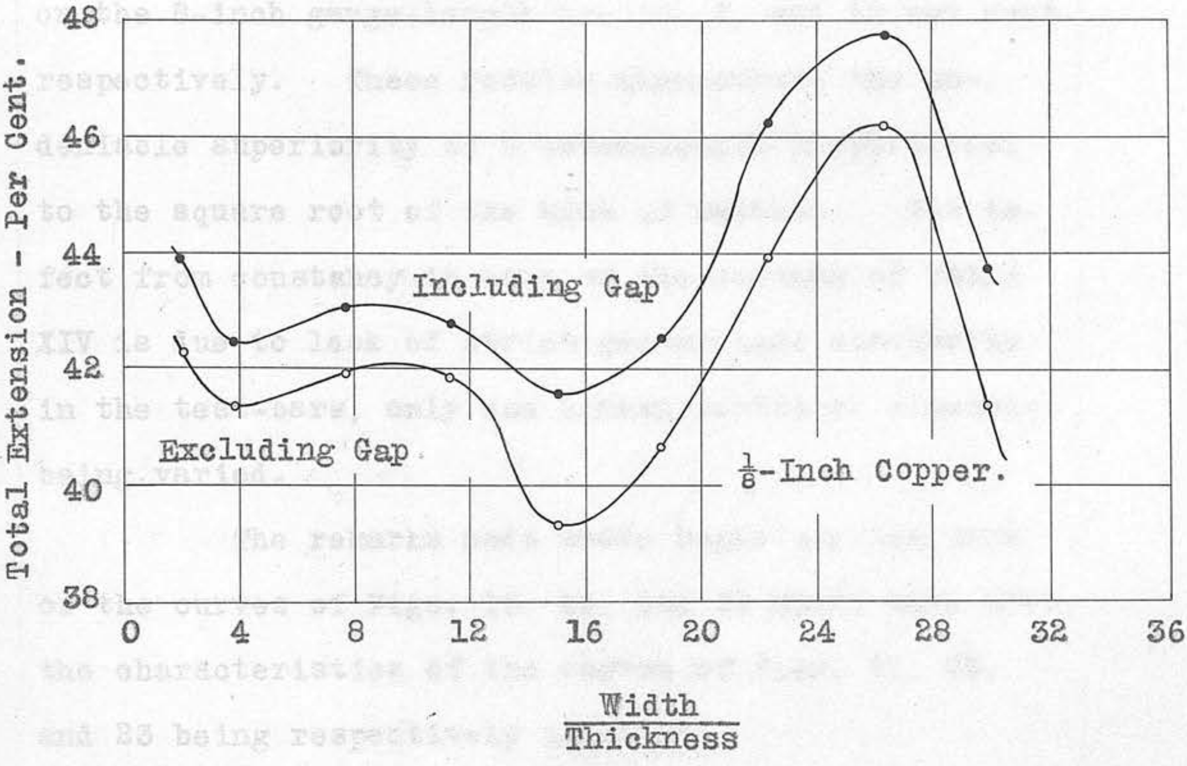

Total Extension - Per Cent.
32
30
28
26
24
22
Including Gap
Excluding Gap
¼-Inch Steel.
0
2
4
6
8
10
12
14
16
18
Width / Thickness

Fig. 21.



34
32
30
28
26
24
Total Extension - Per Cent.
Including Gap
Excluding Gap
$\frac{1}{8}$-Inch Steel.
0
4
8
12
16
20
24
28
32
36
Width / Thickness

Fig.22.



Total Extension - Per Cent.
48
46
44
42
40
38
0
4
8
12
16
20
24
28
32
36
Width / Thickness
Including Gap
Excluding Gap
$\frac{1}{8}$-Inch Copper.

Fig.23.

particular gauge-length is not very great, the maximum amount of variation or the difference between the extreme values being in the present cases approximately 2 per cent. for the thicker steel, 3 per cent. for the thinner steel, and 7 per cent. for the copper. It will be recalled that the corresponding variations on the 8-inch gauge-length are 10, 7, and 13 per cent. respectively. These results demonstrate the undeniable superiority of a datum-length proportional to the square root of the area of section. The defect from constancy in each of the columns of Table XIV is due to lack of strict geometrical similarity in the test-bars, only one linear sectional dimension being varied.

The remarks made above regarding the form of the curves of Figs. 18, 19, and 20 apply here also, the characteristics of the curves of Figs. 21, 22, and 23 being respectively identical.

Excluding those for the smallest gauge-lengths, all these curves have certain common features which markedly differentiate them from the corresponding curves for fixed lengths. If it is the case that a periodic element is present, the axis of reference, oblique in the case of extension on fixed lengths, becomes practically horizontal in the present instance. Comparing the 11.3 $\sqrt{\text{area}}$ curve of Fig. 21 with its 8-inch analogue (Fig. 15), it is observed that the most important change in form is the depression between the values 7 and 12 of the abscissa. Postulating the harmonic relation, it would appear that the factor which ex hypothesi is able to bring down to the horizontal the upper curve for the 8-inch gauge-length, is in the present case capable of depressing the corresponding curve to its initial level. The effect of relative decrease of length is thus more than neutralised over the critical range. These remarks, of course, apply generally to the curves of this set.

Referring to Fig. 19B, the curves for the specimens cut from the $\frac{1}{8}$-inch steel plate, it is observed that the slight fall in extension with increase in width of test-piece is preceded by what is probably a very rapid rise. A three-termed equation gives what appears to be the law of variation, but for simplicity the results in the case of this high-grade metal have been treated on two straight-line bases - the one horizontal, the other oblique. On the former - that of constancy - the departures of the observed values from their mean have been determined for all three metals, and in the case of the gauge-length of 11.3 $\sqrt{\text{area}}$ are given in Table XIVA. Considering the very wide ranges of the ratio $\frac{\text{width}}{\text{thickness}}$, the figures in the 'Difference' columns for the two steels might be regarded as passable, the average values of the residuals, irrespective of sign, being 0.7 per cent. for the thicker and 0.5 per cent. for the thinner plate and the corresponding algebraic sums -0.2 and -0.1 per cent. The differences for the copper are, however, wholly unsatisfactory.

Treatment on the oblique basis has been confined to the results obtained from the thinner steel specimens and gives, as would be expected, a somewhat better distribution. The equation

$$e = 30.71 - 0.055 \frac{W}{T}$$

was found, and from it were calculated the values of extension listed in Table XIVB. In this case the arithmetic mean of the differences is 0.5 per cent. and the algebraic sum zero.

It has been stated earlier in this Article that the Law of Similarity may be extended to include the comparison of extensions obtained from non-similar, but compact, specimens of the same material provided the elongations be measured on a gauge-length

TABLE XIVA.

Calculated Values of Extension on 11.3 $\sqrt{Area}$.
(Horizontal or Mean.)

Nominal Width. Inches.	Extension per cent.								
	$\frac{1}{4}$-Inch Steel.			$\frac{1}{8}$-Inch Steel.			$\frac{1}{8}$-Inch Copper.		
	Observed.	Calculated. (Mean)	Difference.	Observed.	Calculated. (Mean)	Difference.	Observed.	Calculated. (Mean)	Difference.
$\frac{1}{4}$	-	-	-	29.9	29.9	$\pm$0.0	42.3	42.0	+0.3
$\frac{1}{2}$	27.6	28.3	-0.7	30.7	"	+0.8	41.3	"	-0.7
1	28.1	"	-0.2	31.1	"	+1.2	41.9	"	-0.1
$1\frac{1}{2}$	29.2	"	+0.9	29.4	"	-0.5	41.8	"	-0.2
2	29.5	"	+1.2	30.3	"	+0.4	39.3	"	-2.7
$2\frac{1}{2}$	28.5	"	+0.2	29.7	"	-0.2	40.6	"	-1.4
3	27.3	"	-1.0	29.9	"	$\pm$0.0	43.9	"	+1.9
$3\frac{1}{2}$	27.3	"	-1.0	28.4	"	-1.5	46.1	"	+4.1
4	28.7	"	+0.4	29.6	"	-0.3	41.3	"	-0.7

TABLE XIVB.

$\frac{1}{8}$-Inch Steel - Calculated Values of Extension on $11.3\sqrt{\text{Area}}$.
(Oblique)

Nominal Width.	Extension per cent.		
Inches.	Observed.	Calculated.	Difference.
$\frac{1}{4}$	29.9	30.6	-0.7
$\frac{1}{2}$	30.7	30.5	+0.2
1	31.1	30.3	+0.8
$1\frac{1}{2}$	29.4	30.1	-0.7
2	30.3	29.9	+0.4
$2\frac{1}{2}$	29.7	29.7	$\pm$0.0
3	29.9	29.5	+0.4
$3\frac{1}{2}$	28.4	29.3	-0.9
4	29.6	29.1	+0.5

of the type under discussion. The results now set forth under the present head give an idea of how far this assumption of virtual similarity is legitimate in the case of plate specimens of which only one transverse dimension is varied, and that over a considerable range. The conclusion is that, in the case of uniform material, width of specimen has very little influence on the percentage of elongation provided the coefficient of $\sqrt{\text{area}}$ be reasonably large. It follows that in such circumstances extensions are practically comparable. Considering the obvious merits and superiority of this class of gauge-length, it is regrettable that in commercial testing in this country the suggestion of its general adoption does not meet with more favour.

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8. Extension on Lengths Proportional to the Width.

It was indicated in Art. 6 that, ceteris paribus, there is proportionally less extension in a wide bar than in a narrow one. The truth of this may be proved in several ways, one method being to determine the extension on various lengths each of which is proportional to the width of the specimen. This has been done for the three sets of bars under consideration; but as the thickness of each plate was practically uniform, the gauge-lengths were made proportional to the width divided by the thickness. The resulting values are listed in Tables XV, XVI, and XVII and shown plotted against the ratio $\frac{\text{width}}{\text{thickness}}$ in Figs. 24, 25, and 26. To keep the gauge-length within the available range and to preserve clarity of diagram, the number of widths discussed is limited to three. The values of the extension for the several lengths were read directly from the large-scale extension-length curves, those for the two or three lengths lying slightly under $\frac{1}{2}$ inch being found by extrapolation. Owing to variation in the position of the fracture within the first datum unit, these latter - in fact, all the values for the smallest lengths - are subject to error.

TABLE XV.

$\frac{1}{4}$-Inch Steel - Extension on Lengths Proportional to the Width.

Nominal Width. Inches.	Extension per cent. on a Length in Inches equal to		
	$\frac{1}{4}\,\frac{\text{Width}}{\text{Thickness}}$.	$\frac{1}{2}\,\frac{\text{Width}}{\text{Thickness}}$.	$\frac{\text{Width}}{\text{Thickness}}$.
$\frac{1}{2}$	56.8	46.5	37.0
1	50.6	40.5	31.3
$1\frac{1}{2}$	49.9	39.9	31.6
2	49.0	38.4	30.2
$2\frac{1}{2}$	47.5	35.8	28.1
3	43.7	33.6	26.1
$3\frac{1}{2}$	42.8	32.5	24.9
4	42.4	32.9	25.6

TABLE XVI.

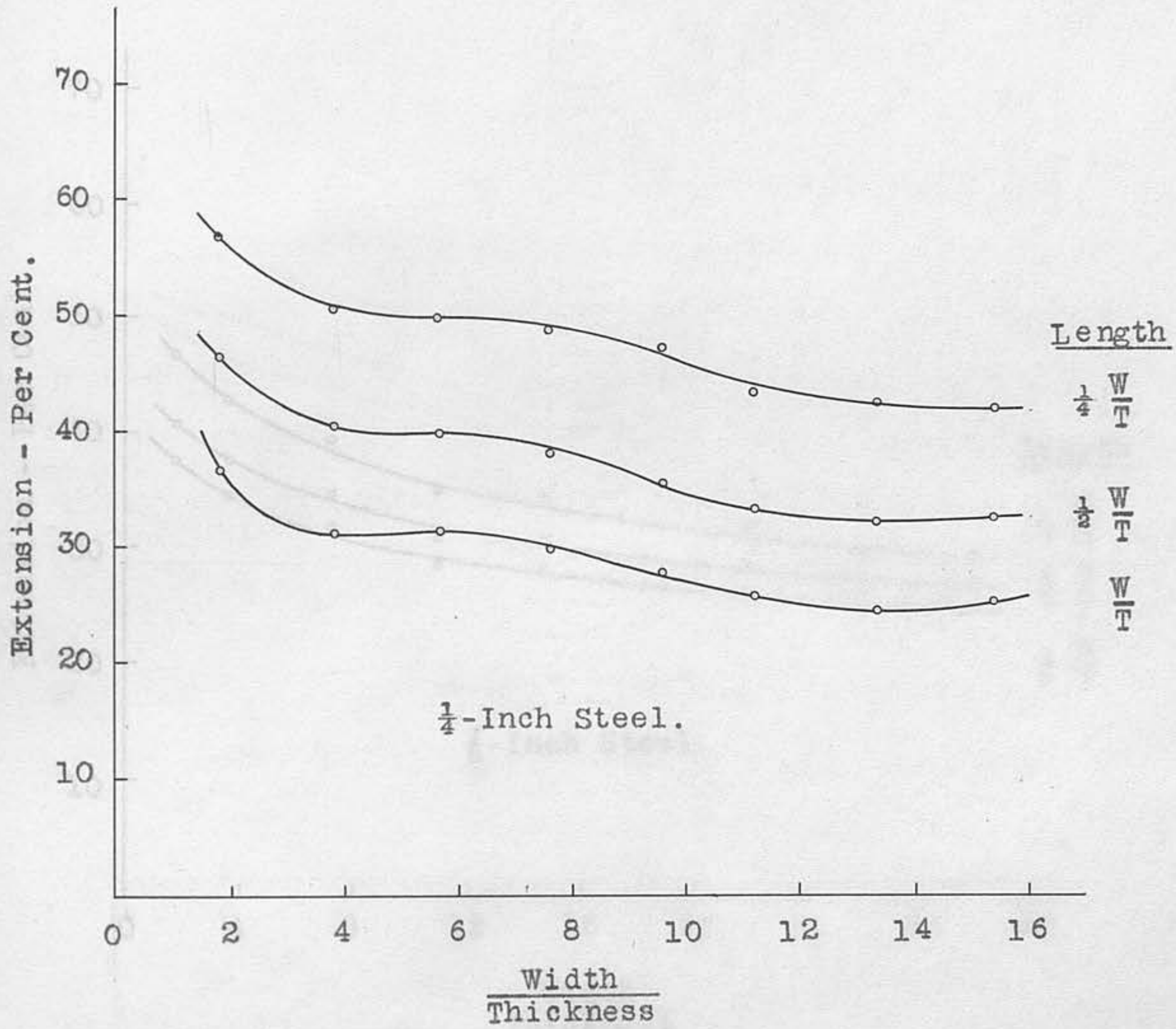
$\frac{1}{8}$-Inch Steel - Extension on Lengths Proportional to the Width.

Nominal Width. Inches.	Extension per cent. on a Length in Inches equal to		
	$\frac{1}{4}\,\frac{\text{Width}}{\text{Thickness}}$.	$\frac{3}{8}\,\frac{\text{Width}}{\text{Thickness}}$.	$\frac{1}{2}\,\frac{\text{Width}}{\text{Thickness}}$.
$\frac{1}{4}$	46.7	40.9	37.7
$\frac{1}{2}$	43.1	37.8	34.7
1	39.5	34.8	32.2
$1\frac{1}{2}$	35.2	31.1	28.9
2	34.6	31.0	28.6
$2\frac{1}{2}$	32.4	29.5	27.1
3	32.2	28.9	26.8
$3\frac{1}{2}$	29.7	26.8	24.7
4	30.2	27.6	25.5

TABLE XVII.

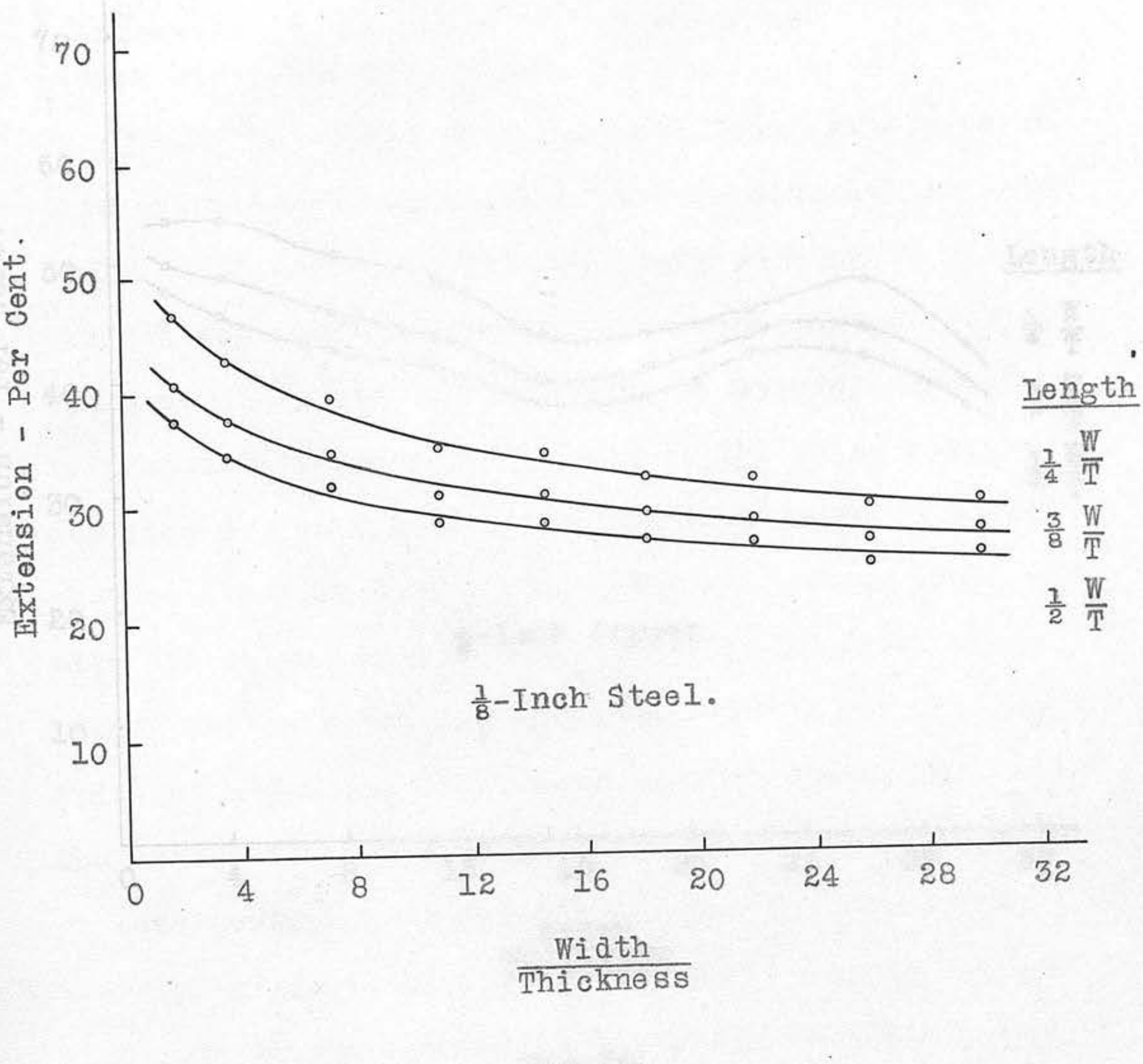
$\frac{1}{8}$-Inch Copper - Extension on Lengths Proportional to the Width.

Nominal Width. Inches.	Extension per cent. on a Length in Inches equal to		
	$\frac{1}{4}\ \frac{\text{Width}}{\text{Thickness}}$.	$\frac{3}{8}\ \frac{\text{Width}}{\text{Thickness}}$.	$\frac{1}{2}\ \frac{\text{Width}}{\text{Thickness}}$.
$\frac{1}{4}$	53.9	50.2	47.9
$\frac{1}{2}$	53.8	48.9	45.8
1	51.0	45.7	42.6
$1\frac{1}{2}$	48.5	43.5	40.8
2	43.3	39.6	37.5
$2\frac{1}{2}$	43.5	40.2	38.3
3	45.3	43.4	41.8
$3\frac{1}{2}$	47.9	43.7	40.9
4	42.1	39.1	37.0



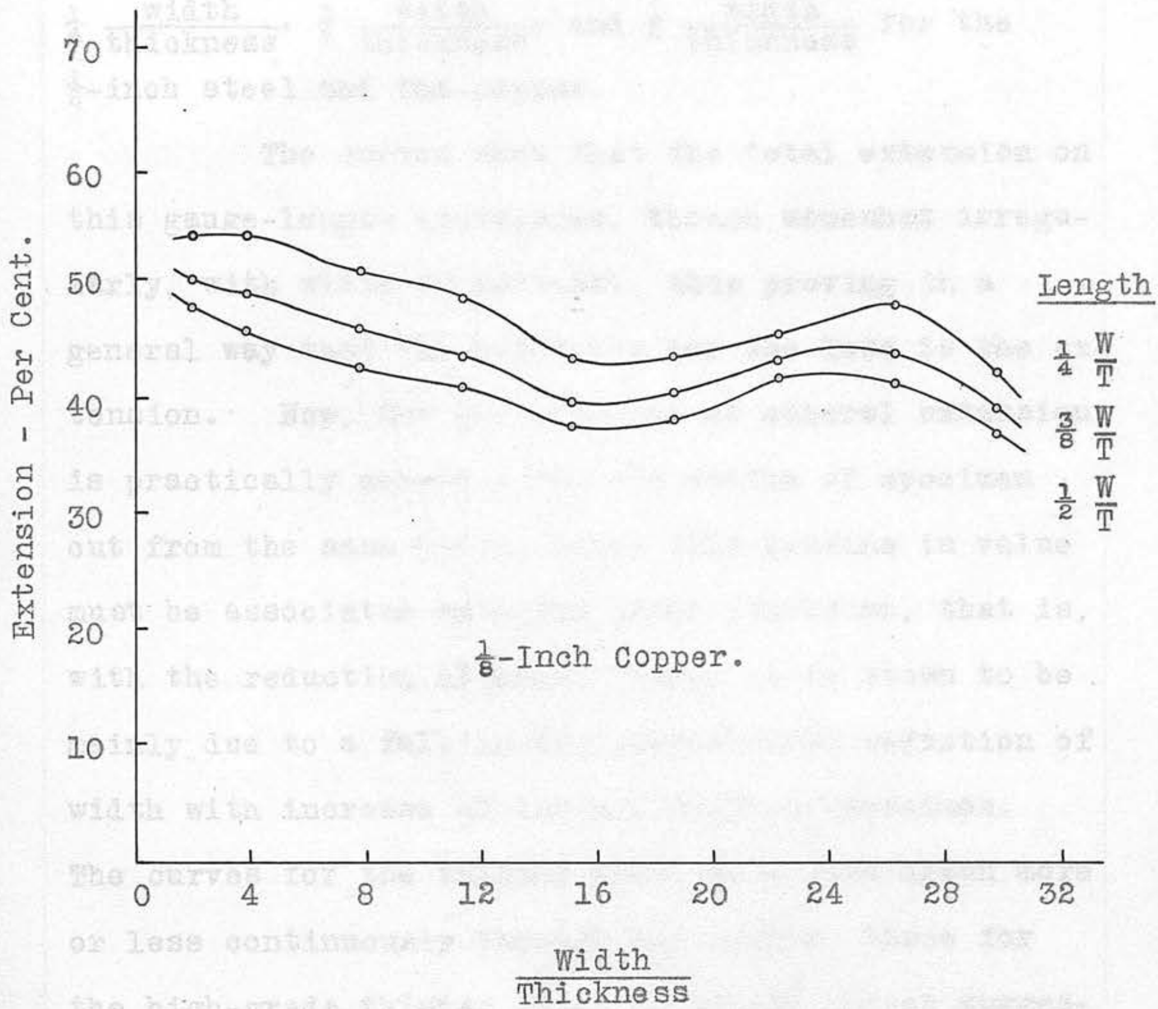
70
60
50
40
30
20
10
Extension - Per Cent.
0
2
4
6
8
10
12
14
16
Width / Thickness
Length
$\frac{1}{4}\frac{W}{T}$
$\frac{1}{2}\frac{W}{T}$
$\frac{W}{T}$
$\frac{1}{4}$-Inch Steel.

Fig.24.



Extension - Per Cent.
70
60
50
40
30
20
10
0
4
8
12
16
20
24
28
32
Width / Thickness
Length
$\frac{1}{4}\frac{W}{T}$
$\frac{3}{8}\frac{W}{T}$
$\frac{1}{2}\frac{W}{T}$
$\frac{1}{8}$-Inch Steel.

Fig.25.



Extension - Per Cent.
70
60
50
40
30
20
10
0
4
8
12
16
20
24
28
32
Width / Thickness
Length
1/4 W/T
3/8 W/T
1/2 W/T
1/8-Inch Copper.

Fig.26.

Table XV gives the percentages of elongation on lengths (in inches) equal to $\frac{1}{4}\frac{\text{width}}{\text{thickness}}$, $\frac{1}{2}\frac{\text{width}}{\text{thickness}}$, and $\frac{\text{width}}{\text{thickness}}$ for the $\frac{1}{4}$ inch steel, and Tables XVI and XVII those on lengths equal to $\frac{1}{4}\frac{\text{width}}{\text{thickness}}$, $\frac{3}{8}\frac{\text{width}}{\text{thickness}}$, and $\frac{1}{2}\frac{\text{width}}{\text{thickness}}$ for the $\frac{1}{8}$-inch steel and the copper.

The curves show that the total extension on this gauge-length diminishes, though somewhat irregularly, with width of test-bar, thus proving in a general way that the wider the bar the less is the extension. Now, the proportional or general extension is practically constant for all widths of specimen cut from the same plate, hence this decline in value must be associated with the local extension, that is, with the reduction of area: later it is shown to be mainly due to a fall in the proportional reduction of width with increase of initial width of specimen. The curves for the thicker steel have been drawn more or less continuously through the points, those for the high-grade thinner steel as smooth curves suggesting some simple relation between the variables, and those for the copper directly through the points. The results obtained from the two steels appear to be quite satisfactory, but those from the copper are, as usual, somewhat erratic, the exceptionally large extensions around the $3\frac{1}{2}$-inch width of the latter still persisting.

Confining the attention to the thinner steel, it is to be noted that if the curves had been drawn continuously through the points, maxima would have been apparent over the values $7\frac{1}{2}$, 15, $22\frac{1}{2}$, and probably 30, of the abscissa, which values are identical with those revealed by the curves of Figs. 13 and 13A for the maxima of extension on fixed lengths. The equations of curves such as those of Fig. 25 are not deducible from the extension-length relation of Art. 6, $e = \frac{a}{\sqrt{L}} + b$, by putting $L = K\frac{W}{T}$, for the simple reason that although $\underline{b}$ appears to be a constant for the material, $\underline{a}$ varies with the width of specimen and is constant only for a given width. As a matter of fact, the curves under consideration are not hyperbolic at all, the experimental results being best fitted by an exponential or a logarithmic equation.

Following the assumption that the relation is non-periodic, trials made with various straightening functions suggested two types of equation each of which gave curves fitting the experimental results with a degree of accuracy sufficient for all practical purposes. These are:-

(1) the exponential equation

$$e = a\epsilon^{b\frac{W}{T}} + c$$

in which ϵ is Napier's base and a, b, and c are constants, the second being negative and fractional, and

(2) the simpler inverse logarithmic expression

$$e = a + b \log \frac{W}{T}$$

in which a and b are constants, the latter being negative.

To determine the values of the constants the usual procedure with the Method of Averages was carried out; and the following empirical formulae were obtained: -

(1) <u>Exponential</u>

$$L = \frac{1}{4}\frac{W}{T}. \qquad e = 21.03\epsilon^{-0.0930\frac{W}{T}} + 28.57$$

$$L = \frac{3W}{8T}. \qquad e = 16.28\epsilon^{-0.0824\frac{W}{T}} + 25.70$$

$$L = \frac{1W}{2T}. \qquad e = 15.25\epsilon^{-0.0852\frac{W}{T}} + 23.80$$

(2) <u>Logarithmic</u>

$$L = \frac{1W}{4T}. \qquad e = 51.21-14.63 \log \frac{W}{T}$$

$$L = \frac{3}{8}\frac{W}{T}. \quad e = 44.19 - 11.64 \log \frac{W}{T}$$

$$L = \frac{1}{2}\frac{W}{T}. \quad e = 40.80 - 10.76 \log \frac{W}{T}$$

The degree of closeness between the observed values and those calculated from the preceding formulae is seen from the entries in the 'Difference' columns of Tables XVIA and XVIB to be very satisfactory. The average value of the residuals, irrespective of sign, is in each case practically negligible, the mean difference of all being of the order one-half of one per cent; the algebraic sum is in two cases zero; and the plus and minus signs are well distributed. There is not much to choose between the two relations as regards 'fit,' but on the score of simplicity the logarithmic equation would naturally be employed, in which case it may be said that the percentage of elongation on a gauge-length proportional to the ratio $\frac{\text{width}}{\text{thickness}}$ plus a multiple of the logarithm of the ratio itself is constant.

If the quasi-harmonic variation be taken into account, the addition to the above equations of another term which is a simple function of $\sin \pi \frac{W}{T}$ would probably result in still lower residuals, but with the data available such procedure would not be justified.

TABLE XVIA.

$\frac{1}{8}$-Inch Steel - Calculated Values of Extension on Lengths Proportional to the Width.

(Exponential)

Nominal Width. Inches.	Extension per cent. on a Length in Inches equal to								
	$\frac{1}{4}\frac{\text{Width}}{\text{Thickness}}$.			$\frac{3}{8}\frac{\text{Width}}{\text{Thickness}}$.			$\frac{1}{2}\frac{\text{Width}}{\text{Thickness}}$.		
	Observed	Calculated	Difference	Observed	Calculated	Difference	Observed	Calculated	Difference
$\frac{1}{4}$	46.7	46.3	+0.4	40.9	39.7	+1.2	37.7	36.9	+0.8
$\frac{1}{2}$	43.1	43.5	-0.4	37.8	37.7	+0.1	34.7	35.0	-0.3
1	39.5	39.2	+0.3	34.8	34.6	+0.2	32.2	32.0	+0.2
$1\frac{1}{2}$	35.2	36.1	-0.9	31.1	32.3	-1.2	28.9	29.8	-0.9
2	34.6	34.0	+0.6	31.0	30.6	+0.4	28.6	28.2	+0.4
$2\frac{1}{2}$	32.4	32.5	-0.1	29.5	29.3	+0.2	27.1	27.0	+0.1
3	32.2	31.3	+0.9	28.9	28.4	+0.5	26.8	26.1	+0.7
$3\frac{1}{2}$	29.7	30.4	-0.7	26.8	27.6	-0.8	24.7	25.5	-0.8
4	30.2	29.9	+0.3	27.6	27.1	+0.5	25.5	25.0	+0.5

TABLE XVIB.

$\frac{1}{8}$-Inch Steel - Calculated Values of Extension on Lengths Proportional to the Width.

(Logarithmic)

Nominal Width. Inches.	Extension per cent. on a Length in Inches equal to								
	$\frac{1}{4}\frac{\text{Width}}{\text{Thickness}}$.			$\frac{3}{8}\frac{\text{Width}}{\text{Thickness}}$.			$\frac{1}{2}\frac{\text{Width}}{\text{Thickness}}$.		
	Observed	Calculated	Difference	Observed	Calculated	Difference	Observed	Calculated	Difference
$\frac{1}{4}$	46.7	47.4	-0.7	40.9	41.2	-0.3	37.7	38.0	-0.3
$\frac{1}{2}$	43.1	43.0	+0.1	37.8	37.6	+0.2	34.7	34.7	±0.0
1	39.5	38.5	+1.0	34.8	34.1	+0.7	32.2	31.5	+0.7
$1\frac{1}{2}$	35.2	36.0	-0.8	31.1	32.1	-1.0	28.9	29.6	-0.7
2	34.6	34.2	+0.4	31.0	30.6	+0.4	28.6	28.3	+0.3
$2\frac{1}{2}$	32.4	32.8	-0.4	29.5	29.5	±0.0	27.1	27.2	-0.1
3	32.2	31.6	+0.6	28.9	28.6	+0.3	26.8	26.4	+0.4
$3\frac{1}{2}$	29.7	30.5	-0.8	26.8	27.7	-0.9	24.7	25.6	-0.9
4	30.2	29.6	+0.6	27.6	27.0	+0.6	25.5	24.9	+0.6

The magnitude of the variation of extension as shown by the entries in the several columns of Tables XV, XVI, and XVII is such as to indicate that comparable results are not to be expected from this type of gauge-length, the influence of the width of the specimen being too strongly marked.

9. <u>Extension on Lengths Proportional to Area/Perimeter.</u>

Some time ago it occurred to the writer that an investigation of the extension on gauge-lengths proportional to the ratio of the cross-sectional area to the perimeter of the test-bar might yield results which, if not comparable, would at least be of considerable interest. As regards 'dimension,' this ratio - suggestive of the <u>Hydraulic Mean Depth</u> - is obviously of the first degree in <u>Length.</u> The extension-length curves derived from the figures recorded in Tables VI, VII, and VIII have afforded the opportunity of making this enquiry. The result is the present Article which, accordingly, deals with the influence of the width of the specimen on the extension on lengths proportional to the ratio $\frac{\text{area}}{\text{perimeter}}$. The magnitude of this ratio in the present series of experiments is so small that comparatively large multiples have had to be used to bring the gauge-lengths up to reasonable absolute dimensions, that is, to values approximating to those used in practice.

The extensions on lengths (in inches) equal to 40 $\frac{\text{area}}{\text{perimeter}}$, 80 $\frac{\text{area}}{\text{perimeter}}$, and 120 $\frac{\text{area}}{\text{perimeter}}$ for the $\frac{1}{4}$-inch steel and on lengths equal to

$80 \frac{\text{area}}{\text{perimeter}}$, $160 \frac{\text{area}}{\text{perimeter}}$, and $240 \frac{\text{area}}{\text{perimeter}}$ for the $\frac{1}{8}$-inch steel and the copper are set forth in Tables XVIII, XIX, and XX respectively; and the graphs corresponding to these figures are shown in Figs. 27, 28, and 29, the base, as previously, being the ratio $\frac{\text{width}}{\text{thickness}}$ of the piece.

The curves, like those of Figs. 12, 13, and 14 (Art. 6) may be regarded as simply continuous or progressively harmonic. On the former basis, the general shape suggests the parabola as the graphical expression of the physical law underlying the observed quantities: this is particularly well brought out by the curves of Fig. 28, the values of the extension for this high-class material permitting the graphs to be drawn as shown. But, in this case too, an equally good 'fit' within the experimental limits is obtained on applying the simple logarithmic equation previously employed, in which case, of course, correctness of form is taken as subordinate to simplicity and convenience. It is to be noted, as in all the preceding work on extension, that the influence of the width of specimen becomes less marked as the gauge-length is increased.

The most satisfactory empirical formulae were derived from: -

TABLE XVIII.

$\frac{1}{4}$-Inch Steel - Extension on Lengths Proportional to $\frac{\text{Area}}{\text{Perimeter}}$.

Nominal Width. Inches.	$\frac{\text{Area}}{\text{Perimeter}}$.	Extension per cent. on a Length in Inches equal to		
		$40\frac{\text{Area}}{\text{Perimeter}}$.	$80\frac{\text{Area}}{\text{Perimeter}}$.	$120\frac{\text{Area}}{\text{Perimeter}}$.
$\frac{1}{2}$	0.0814	29.3	24.0	21.7
1	0.1006	31.0	25.2	23.3
$1\frac{1}{2}$	0.1087	34.5	27.3	24.3
2	0.1135	35.9	28.1	24.8
$2\frac{1}{2}$	0.1172	36.2	28.2	24.7
3	0.1193	35.8	27.5	23.8
$3\frac{1}{2}$	0.1205	37.0	28.5	24.4
4	0.1207	39.0	30.4	26.0

TABLE XIX.

$\frac{1}{8}$-Inch Steel - Extension on Lengths Proportional to $\frac{\text{Area}}{\text{Perimeter}}$.

Nominal Width. Inches.	$\frac{\text{Area}}{\text{Perimeter}}$.	Extension per cent. on a Length in Inches equal to		
		$80\frac{\text{Area}}{\text{Perimeter}}$.	$160\frac{\text{Area}}{\text{Perimeter}}$.	$240\frac{\text{Area}}{\text{Perimeter}}$.
$\frac{1}{4}$	0.0438	26.3	22.9	21.7
$\frac{1}{2}$	0.0530	28.1	24.2	22.1
1	0.0599	30.2	25.5	23.3
$1\frac{1}{2}$	0.0623	29.6	25.1	22.5
2	0.0641	31.6	26.2	23.6
$2\frac{1}{2}$	0.0649	31.4	26.2	23.2
3	0.0650	32.7	27.1	24.3
$3\frac{1}{2}$	0.0650	31.8	26.3	23.4
4	0.0648	33.1	28.2	25.2

TABLE XX.

$\frac{1}{8}$-Inch Copper - Extension on Lengths Proportional to $\frac{\text{Area}}{\text{Perimeter}}$.

Nominal Width. Inches.	$\frac{\text{Area}}{\text{Perimeter}}$.	Extension per cent. on a Length in Inches equal to		
		$80\frac{\text{Area}}{\text{Perimeter}}$.	$160\frac{\text{Area}}{\text{Perimeter}}$.	$240\frac{\text{Area}}{\text{Perimeter}}$.
$\frac{1}{4}$	0.0432	39.3	34.0	31.6
$\frac{1}{2}$	0.0513	38.9	35.2	33.8
1	0.0575	40.7	36.8	35.1
$1\frac{1}{2}$	0.0606	42.1	37.4	35.4
2	0.0623	40.7	36.6	34.2
$2\frac{1}{2}$	0.0640	42.7	37.9	36.5
3	0.0641	45.9	42.5	39.2
$3\frac{1}{2}$	0.0641	50.8	43.3	39.4
4	0.0648	45.5	39.6	36.7

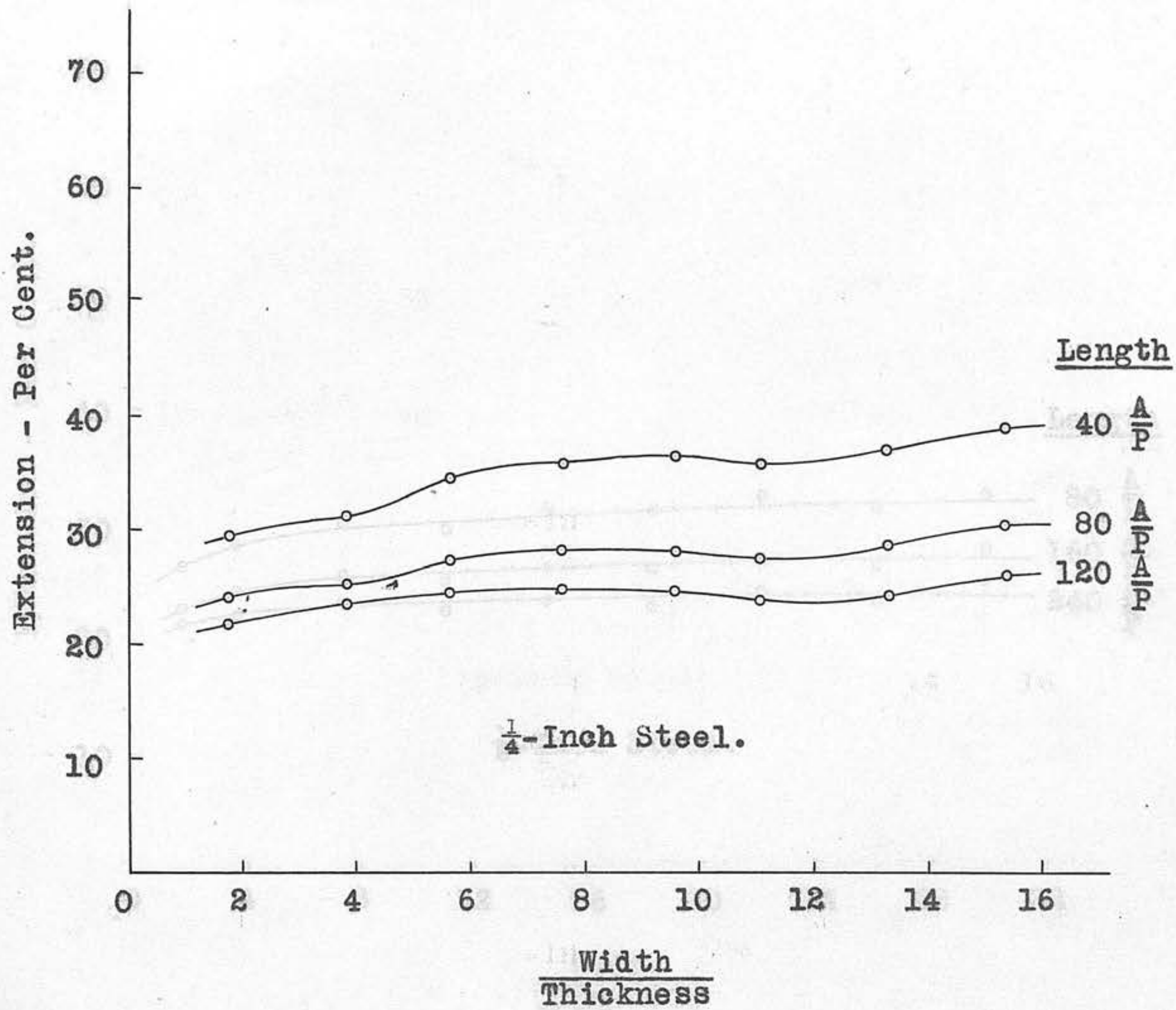

70
60
50
40
30
20
10
Extension - Per Cent.
0
2
4
6
8
10
12
14
16
Width / Thickness
Length
40 A/P
80 A/P
120 A/P
¼-Inch Steel.

Fig.27.

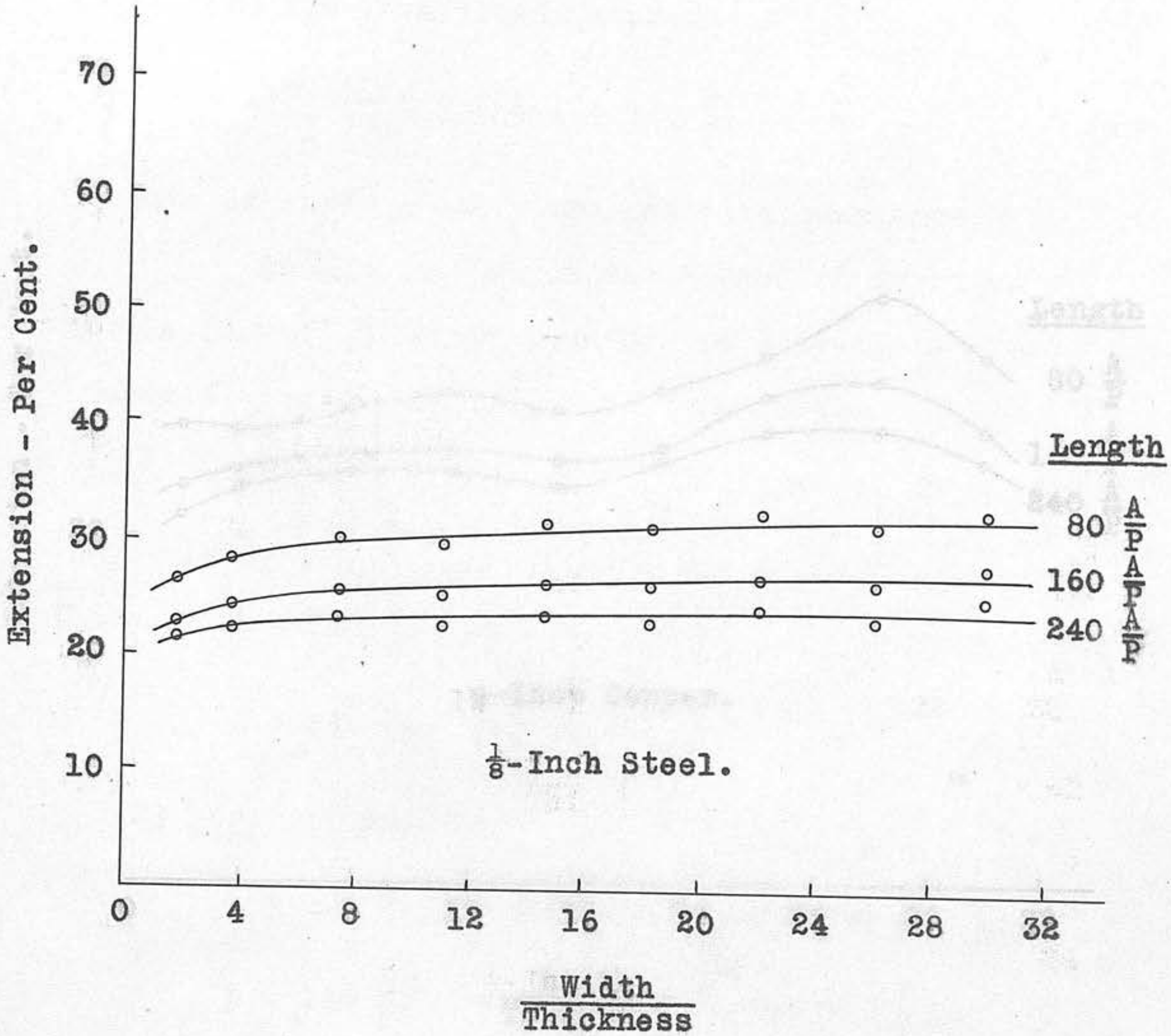
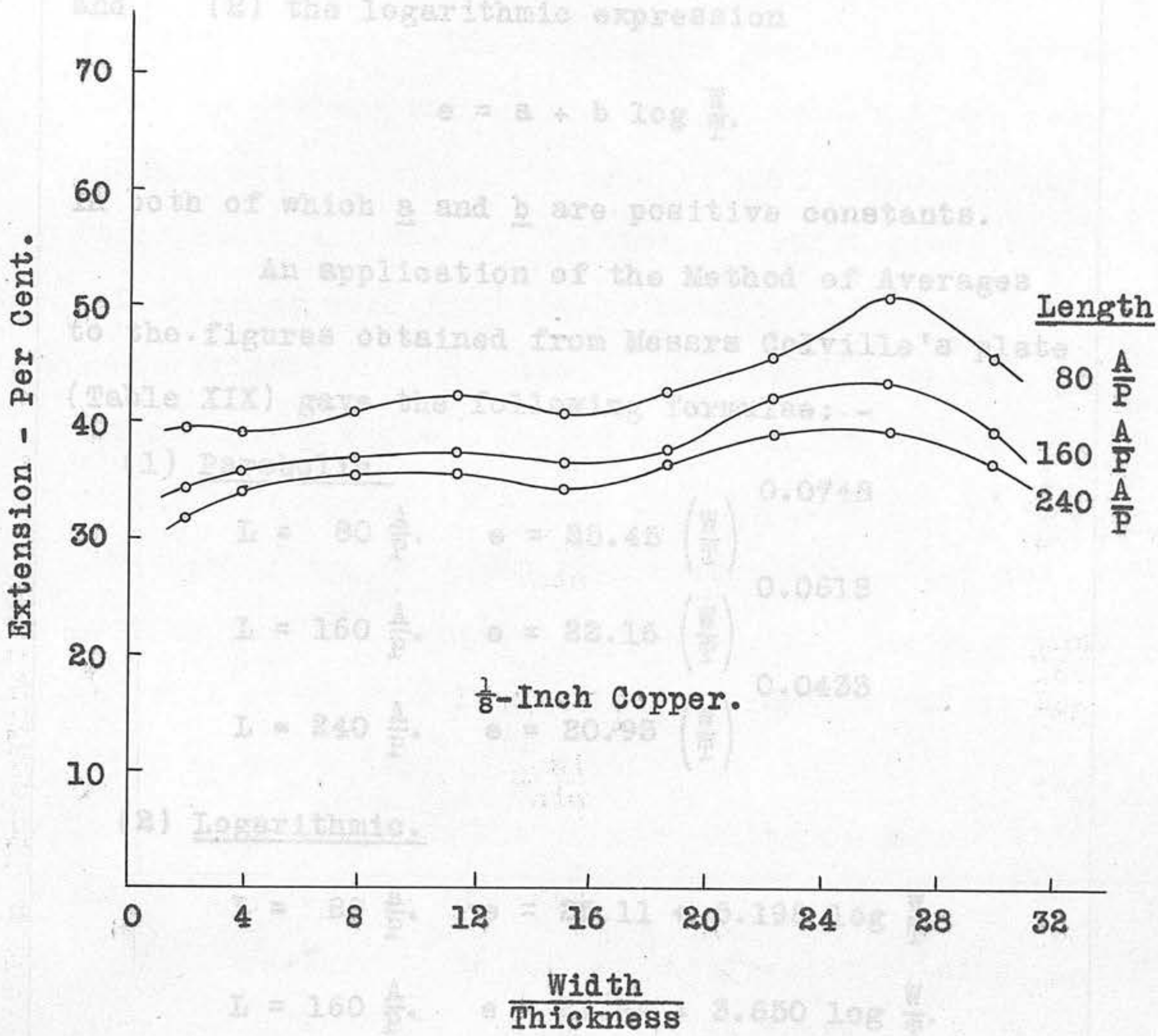

70
60
50
40
30
20
10
0
4
8
12
16
20
24
28
32
Extension - Per Cent.
Length
80 A/P
160 A/P
240 A/P
$\frac{1}{8}$-Inch Steel.
Width / Thickness

Fig.28.



70
60
50
40
30
20
10
0
4
8
12
16
20
24
28
32
Extension - Per Cent.
Width / Thickness
Length
80 A/P
160 A/P
240 A/P
1/8-Inch Copper.

Fig.29.

(1) the parabolic equation

$$e = a\left(\frac{W}{T}\right)^{b}$$

and (2) the logarithmic expression

$$e = a + b \log \frac{W}{T},$$

in both of which <u>a</u> and <u>b</u> are positive constants.

An application of the Method of Averages to the figures obtained from Messrs Colville's plate (Table XIX) gave the following formulae: -

(1) <u>Parabolic.</u>

$$L = 80\,\frac{A}{P}. \quad e = 25.45\left(\frac{W}{T}\right)^{0.0748}$$

$$L = 160\,\frac{A}{P}. \quad e = 22.16\left(\frac{W}{T}\right)^{0.0618}$$

$$L = 240\,\frac{A}{P}. \quad e = 20.93\left(\frac{W}{T}\right)^{0.0433}$$

(2) <u>Logarithmic.</u>

$$L = 80\,\frac{A}{P}. \quad e = 25.11 + 5.198 \log \frac{W}{T}.$$

$$L = 160\,\frac{A}{P}. \quad e = 21.94 + 3.650 \log \frac{W}{T}.$$

$$L = 240\,\frac{A}{P}. \quad e = 20.82 + 2.330 \log \frac{W}{T}.$$

It is observed that in each type of relation the values of both constants fall with increase of gauge-length, and that the corresponding values of the first constant in both types are in approximate agreement. The values of the extension calculated from these equations and their differences from the observed results are listed in Tables XIXA and XIXB. The former, giving the results on the parabolic basis, shows an average value of 0.44 per cent. for the residuals and an arithmetic mean of 0.13 per cent. for their algebraic sums. The latter, which gives the values calculated from the logarithmic formulae, shows slightly better results, the average value of the residuals being 0.42 per cent. and the arithmetic mean of their algebraic sums 0.10 per cent. From a practical point of view, such a degree of closeness must be regarded as quite satisfactory; particularly so, when it is considered that the maximum variation <u>inter se</u> for a set of bars of given width is in general greater than the corresponding residual for the average of the same set.

Referring solely to the results obtained from the thinner plate, it may with reason be deduced that the extension-width relation for gauge-lengths proportional to the ratio $\frac{\text{area}}{\text{perimeter}}$ is approximately parabolic, but that for simplicity and facility in calculation the logarithmic equation may be employed.

TABLE XIXA.

$\frac{1}{8}$-Inch Steel - Calculated Values of Extension on Lengths Proportional to $\frac{\text{Area}}{\text{Perimeter}}$.

(Parabolic).

Nominal Width.	Extension per cent. on a Length in Inches equal to								
Inches.	$80\frac{\text{Area}}{\text{Perimeter}}$.			$160\frac{\text{Area}}{\text{Perimeter}}$.			$240\frac{\text{Area}}{\text{Perimeter}}$.		
	Observed	Calculated	Difference	Observed	Calculated	Difference	Observed	Calculated	Difference
$\frac{1}{4}$	26.3	26.6	-0.3	22.9	23.0	-0.1	21.7	21.5	+0.2
$\frac{1}{2}$	28.1	28.0	+0.1	24.2	24.0	+0.2	22.1	22.1	±0.0
1	30.2	29.5	+0.7	25.5	25.1	+0.4	23.3	22.8	+0.5
$1\frac{1}{2}$	29.6	30.4	-0.8	25.1	25.7	-0.6	22.5	23.2	-0.7
2	31.6	31.1	+0.5	26.2	26.2	±0.0	23.6	23.5	+0.1
$2\frac{1}{2}$	31.4	31.6	-0.2	26.2	26.5	-0.3	23.2	23.7	-0.5
3	32.7	32.1	+0.6	27.1	26.8	+0.3	24.3	23.9	+0.4
$3\frac{1}{2}$	31.8	32.5	-0.7	26.3	27.1	-0.8	23.4	24.1	-0.7
4	33.1	32.8	+0.3	28.2	27.3	+0.9	25.2	24.3	+0.9

TABLE XIXB.

$\frac{1}{8}$-Inch Steel - Calculated Values of Extension on Lengths Proportional to $\frac{\text{Area}}{\text{Perimeter}}$.

(Logarithmic).

Nominal Width. Inches.	Extension per cent. on a Length in Inches equal to								
	$80\frac{\text{Area}}{\text{Perimeter}}$.			$160\frac{\text{Area}}{\text{Perimeter}}$.			$240\frac{\text{Area}}{\text{Perimeter}}$.		
	Observed	Calculated	Difference	Observed	Calculated	Difference	Observed	Calculated	Difference
$\frac{1}{4}$	26.3	26.4	-0.1	22.9	22.9	±0.0	21.7	21.4	+0.3
$\frac{1}{2}$	28.1	28.0	+0.1	24.2	24.0	+0.2	22.1	22.1	±0.0
1	30.2	29.6	+0.6	25.5	25.1	+0.4	23.3	22.8	+0.5
$1\frac{1}{2}$	29.6	30.5	-0.9	25.1	25.7	-0.6	22.5	23.2	-0.7
2	31.6	31.2	+0.4	26.2	26.2	±0.0	23.6	23.5	+0.1
$2\frac{1}{2}$	31.4	31.7	-0.3	26.2	26.5	-0.3	23.2	23.7	-0.5
3	32.7	32.1	+0.6	27.1	26.8	+0.3	24.3	24.0	+0.3
$3\frac{1}{2}$	31.8	32.5	-0.7	26.3	27.1	-0.8	23.4	24.1	-0.7
4	33.1	32.8	+0.3	28.2	27.3	+0.9	25.2	24.3	+0.9

On the assumption that the extension on this type of gauge-length tends asymptotically towards a finite constant value for large values of the width, an attempt was made to fit the exponential curve

$$e = a\epsilon^{b\frac{W}{T}} + c$$

to the results under discussion. In this equation the constants a and b are negative; c, the asymptote, is positive; and ϵ is, as previously, the natural logarithmic base. The values calculated from the resulting empirical formulae were, however, not nearly so satisfactory as those tabulated immediately above and, in consequence, have not been incorporated.

The figures of Tables XVIII, XIX, and XX do not indicate a degree of comparability sufficiently high to merit the suggestion of the use of this class of gauge-length in practice.

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10. Reduction of Area.

The percentage of reduction of area is considered by some engineers to be the true measure of ductility, and is employed by them in preference to the percentage of elongation. The determination of the minimum area of a broken flat test-bar is, however, a matter requiring time and care; and, further, small local defects affect the reduction to a considerable extent. As a matter of fact, the two measures of ductility are definitely related, but the connection holds only when the extension is computed as on an elemental length within that portion of the stricture through which the fracture passes. This result follows from the fact that during plastic deformation the specific weight of a ductile material suffers only an exceedingly slight change. The relation may be expressed as follows -

$$\text{Reduction of area, } r, = \frac{A - A'}{A},$$

where A and A' have their previously assigned meanings and

$$\text{Maximum limiting extension} = \frac{A - A'}{A'}$$

$$= \frac{A - A\,(1 - r)}{A\,(1 - r)}$$

$$= \frac{r}{1 - r}.$$

The mode of fracture of flat test-bars also influences to some extent the magnitude of the ultimate contraction: this matter is discussed in the following Article.

The results obtained under this head are listed in Table XXI and shown plotted in Figs. 30, 31, and 32: they indicate the nature of the variation of this index of quality with change in the width of the specimen. The curves are similar in form to those of Figs. 9, 10, and 11 - a result to be expected a priori.

The reduction of area for the $\frac{1}{4}$-inch steel bars drops rapidly in value as the ratio of width to thickness is increased from 2 to 6, then falls very gently with increase of the ratio to 16. The difference between the extreme values is 10 per cent.

Similarly, the reduction curve for the $\frac{1}{8}$-inch steel specimens shows a marked drop between the ratios 2 and 6, and thereafter a slow but steady decline as the ratio is increased to 32. The values of the contraction in this case are most satisfactory, the maximum variation being only 5 per cent.

The initial drop manifested in the case of the two steels is not so well defined in that of the copper. As the reduced dimensions of a soft copper

TABLE XXI.

Reduction of Area.

Nominal Width. Inches.	Reduction of Area per cent.		
	$\frac{1}{4}$-Inch Steel.	$\frac{1}{8}$-Inch Steel.	$\frac{1}{8}$-Inch Copper.
$\frac{1}{4}$	-	62.1	51.7
$\frac{1}{2}$	64.1	60.4	52.1
1	59.5	59.8	51.7
$1\frac{1}{2}$	56.2	59.2	49.9
2	54.9	58.6	48.2
$2\frac{1}{2}$	56.2	58.1	47.6
3	54.0	57.5	47.1
$3\frac{1}{2}$	56.0	57.0	47.8
4	54.7	57.4	46.2

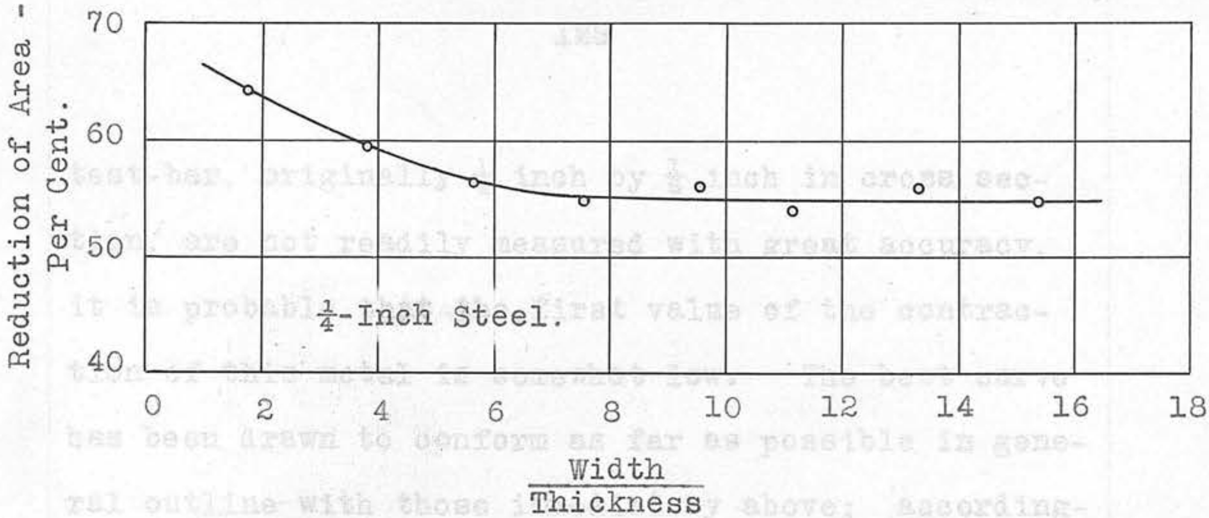

Fig.30.

Fig.31.

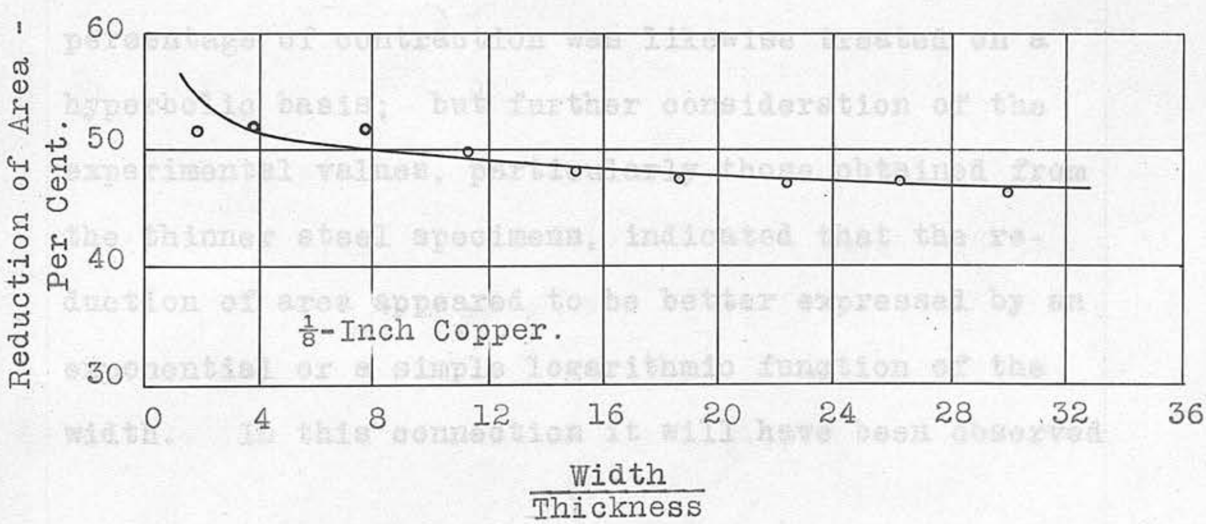

Fig.32.

test-bar, originally $\frac{1}{4}$ inch by $\frac{1}{8}$ inch in cross section, are not readily measured with great accuracy, it is probable that the first value of the contraction of this metal is somewhat low. The best curve has been drawn to conform as far as possible in general outline with those immediately above; accordingly, the first point is practically ignored. The greatest difference of value in this case is 6 per cent. - likewise a very satisfactory result.

The values of the percentage of reduction in all three cases conform to smooth curves with practically sufficient approximation, only a few points deviating therefrom to any degree.

It is to be noted that, with increase in width of bar, the reduction curve appears to tend towards a horizontal asymptote. Following the procedure first employed when dealing with the related curves of extension in the fracture (Art. 6), the percentage of contraction was likewise treated on a hyperbolic basis; but further consideration of the experimental values, particularly those obtained from the thinner steel specimens, indicated that the reduction of area appeared to be better expressed by an exponential or a simple logarithmic function of the width. In this connection it will have been observed that, in the determination of calculated values up to this point, the type of most of the empirical formulae employed has been based on the results obtained from Messrs Colville's high-grade material.

Taking, therefore, the reduction-width relation to be best represented within the experimental range by equations of exponential or of logarithmic type, the following formulae were derived: -

(1) Exponential.

$\frac{1}{4}$-Inch Steel. $$r = 10.06e^{-0.0610\frac{W}{T}} + 50.40$$

$\frac{1}{8}$-Inch Steel. $$r = 6.17e^{-0.0555\frac{W}{T}} + 55.83$$

$\frac{1}{8}$-Inch Copper. $$r = 9.29e^{-0.0905\frac{W}{T}} + 45.90$$

(2) Logarithmic.

$\frac{1}{4}$-Inch Steel. $$r = 63.18 - 7.330 \log \frac{W}{T}$$

$\frac{1}{8}$-Inch Steel. $$r = 63.32 - 4.239 \log \frac{W}{T}$$

$\frac{1}{8}$-Inch Copper. $$r = 55.54 - 6.074 \log \frac{W}{T}.$$

From these expressions the computed values of the percentage of reduction of area listed in Tables XXIA and XXIB have been obtained. In the case of the thicker steel the residuals, although high in both Tables, are roughly of equal magnitude, and the

TABLE XXIA.

Calculated Values of Reduction of Area.

(Exponential)

Nominal Width. Inches.	Reduction of Area per cent.								
	$\frac{1}{4}$-Inch Steel.			$\frac{1}{8}$-Inch Steel.			$\frac{1}{8}$-Inch Copper.		
	Observed.	Calculated.	Difference.	Observed.	Calculated.	Difference.	Observed.	Calculated.	Difference.
$\frac{1}{4}$	-	-	-	62.1	61.4	+0.7	51.7	53.7	-2.0
$\frac{1}{2}$	64.1	<u>59.4</u>	<u>+4.7</u>	60.4	60.9	-0.5	52.1	52.5	-0.4
1	59.5	58.4	+1.1	59.8	59.9	-0.1	51.7	50.5	+1.2
$1\frac{1}{2}$	56.2	57.5	-1.3	59.2	59.2	±0.0	49.9	49.3	+0.6
2	54.9	56.7	-1.8	58.6	58.6	±0.0	48.2	48.3	-0.1
$2\frac{1}{2}$	56.2	56.0	+0.2	58.1	58.1	±0.0	47.6	47.6	±0.0
3	54.0	55.5	-1.5	57.5	57.6	-0.1	47.1	47.1	±0.0
$3\frac{1}{2}$	56.0	54.9	+1.1	57.0	57.3	-0.3	47.8	46.8	+1.0
4	54.7	54.3	+0.4	57.4	57.0	+0.4	46.2	46.5	-0.3

TABLE XXIB.

Calculated Values of Reduction of Area.

(Logarithmic)

Nominal Width. Inches.	Reduction of Area per cent.								
	$\frac{1}{4}$-Inch Steel.			$\frac{1}{8}$-Inch Steel.			$\frac{1}{8}$-Inch Copper.		
	Observed.	Calculated.	Difference.	Observed.	Calculated.	Difference.	Observed.	Calculated.	Difference.
$\frac{1}{4}$	-	-	-	62.1	62.2	-0.1	51.7	53.8	-2.1
$\frac{1}{2}$	64.1	61.4	+2.7	60.4	60.9	-0.5	52.1	52.1	±0.0
1	59.5	58.9	+0.6	59.8	59.7	+0.1	51.7	50.2	+1.5
$1\frac{1}{2}$	56.2	57.7	-1.5	59.2	58.9	+0.3	49.9	49.2	+0.7
2	54.9	56.7	-1.8	58.6	58.4	+0.2	48.2	48.4	-0.2
$2\frac{1}{2}$	56.2	56.0	+0.2	58.1	58.0	+0.1	47.6	47.8	-0.2
3	54.0	55.5	-1.5	57.5	57.6	-0.1	47.1	47.3	-0.2
$3\frac{1}{2}$	56.0	54.9	+1.1	57.0	57.3	-0.3	47.8	46.9	+0.9
4	54.7	54.5	+0.2	57.4	57.1	+0.3	46.2	46.6	-0.4

plus and minus signs are fairly well distributed. The Dalzell steel, however, gives again the most consistent and trustworthy results, the differences in this case being commercially negligible - in fact, the arithmetic mean of these on both bases is 0.2 per cent. and their algebraic sum + 0.1 per cent. and zero on the first and second respectively. Results such as these are a splendid tribute to the skill of the famous Motherwell firm. It is to be observed that in the logarithmic form the values of the first constant for the two steels - the percentages of reduction of area of square bars of these materials - are almost identical. Further, it may be noted that although the exponential formula is not quite successful towards the ends it gives a perfect fit about the middle of the range, also that although not giving a perfect fit anywhere the logarithmic gives a very even distribution as regards both magnitude and sign throughout the entire range. On the score of simplicity the latter type of relation is obviously to be preferred. With the exception of the first, the residuals obtained in the case of the copper may be regarded as fairly satisfactory, especially when consideration is given to the fact that ordinary copper plate is in no sense a uniform material. For the reason already given it is probable that the calculated values for the first width of this plate are nearer the truth than the experimentally determined figure.

In general, apart entirely from the type of equation selected to represent the relation, the reduction-width curve, like the extension-width curve for fixed lengths, appears to tend towards a finite, constant or asymptotic value.

Incidentally, the fall in the value of the reduction of area with increase in width of specimen is a further proof of the statement made in the theory put forward in Art. 6, namely, that proportionally there is less extension in a wide bar than in a narrow one. Further, as actual measurement shows that the final mean thickness is practically constant for all widths of bar, the decline in the percentage of ultimate contraction is due to a gradual diminution in the proportional reduction of width. It is shown in the following Article that this diminution of reduction with increase of transverse dimension is general and that, in consequence, the thickness of a flat test bar is proportionally drawn down more than the width.

The values of the maximum limiting extension in the fracture, calculated from the reduction of area by the relation given earlier in this Article, are naturally in complete agreement with those given in Table IX.

Consideration of the form of the curves of Figs. 13B and 31 suggests that the sum of the percentage of elongation on one of the larger fixed lengths and the percentage of contraction should be an approximately constant number. The same remark applies to the sum of the extension on the selected lengths proportional to $\frac{\text{area}}{\text{perimeter}}$ and the reduction of area. The result of adding the observed values of the percentage of elongation on 8 inches and on a length of $160 \frac{\text{area}}{\text{perimeter}}$ to the corresponding values of the percentage of reduction of area, all for the thinner steel, are set forth side by side in Table XXII. Apart from the last entry in the fifth column, the figures in each case are in remarkably close agreement, the amount of the maximum deviation from the mean, over the entire experimental range being in both cases of the order $1\frac{1}{2}$ per cent. Obviously, if the extensions on greater lengths be taken, or if the similarly calculated values be employed, still better results will be obtained: in fact, the percentage of maximum deviation is then reduced below unity. It

TABLE XXII.

$\frac{1}{8}$-Inch Steel - Sum of Extension and Reduction of Area.

Nominal Width.	Extension per cent. on		Reduction of Area per cent.	Extension on 8 inches + Reduction of Area.	Extension on $160 \frac{\text{Area}}{\text{Perimeter}}$ + Reduction of Area.
Inches.	8 inches.	$160 \frac{\text{Area}}{\text{Perimeter}}$.			
$\frac{1}{4}$	22.5	22.9	62.1	84.6	85.0
$\frac{1}{2}$	24.6	24.2	60.4	85.0	84.6
1	26.6	25.5	59.8	86.4	85.3
$1\frac{1}{2}$	26.5	25.1	59.2	85.7	84.3
2	27.9	26.2	58.6	86.5	84.8
$2\frac{1}{2}$	28.3	26.2	58.1	86.4	84.3
3	29.1	27.1	57.5	86.6	84.6
$3\frac{1}{2}$	28.1	26.3	57.0	85.1	83.3
4	29.7	28.2	57.4	87.1	85.6

It would therefore appear that in the case of steel plate the sum of the two usual criteria of ductility is almost independent of the width of the specimen. This fact suggests that much of the trouble experienced in comparing the results of tensile tests of flats of the same material but of varying width could be obviated by specifying suitable limits within which the sum of the elongation and the contraction must lie - a procedure which would undoubtedly tend to result in economy of material.

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11. <u>The Final Ratio of Width to Thickness.</u>

As the values of the final transverse dimensions of a rectangular test-piece are to a considerable extent determined by the form of the fracture, it may not be out of place to preface the discussion of this ratio by a few remarks on the phenomena of the ultimate deformation of the flat tensile specimen. Such prefatory remarks might, of course, have been legitimately made in the preceding Article on the reduction of area.

Owing to the thinness of the plates used in these researches it is not proposed to deal generally with fracture from the three-dimensional point of view; but the line of rupture on the face of the bar and the variation of thickness in its neighbourhood will be briefly discussed. This will be followed by a note on incipient stricture and an examination into the nature of the variation of the breadth of the gap with width.

The writer has invariably observed that whenever the initial ratio of width to thickness becomes considerable the tendency is towards the oblique type of fracture. With the thinner metals used in the present investigations, in particular with the steel, the line of rupture on the face passed with increase in width of specimen from the symmetric types 1 and 2 of Fig. 33 through the quasi-symmetric type 3 to the wholly asymmetric type 4. In the case of the widest or 4-inch set of the $\frac{1}{8}$-inch steel bars, the angle between the line of rupture and the scribed centre line on the face was found for each bar to be about 70°. This is shown in Fig. 34 which gives a tracing of the outline and line of rupture of one bar of the set. Specimens of such extreme width relative to their thickness almost invariably break at or about this angle which appears to be that at which the arms of the contractile cross are formed. These depressions have been, and sometimes still are, confused or associated with the lines of Lüders. That the two phenomena are distinct is obvious from consideration of the final angles at which the primary lines of deformation and the contractile grooves lie with respect to the centre line of the bar. A photograph (Fig. 35) taken from a piece of $\frac{1}{8}$-inch mild steel hoop (on which metal the research was originally begun) shows the bar just before rupture with one arm of the cross fully developed and traces of the planes of slipping still persisting. The difference of inclination to the direction of

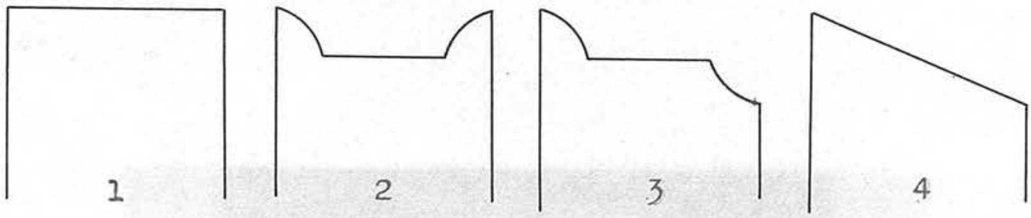

Fig. 33.

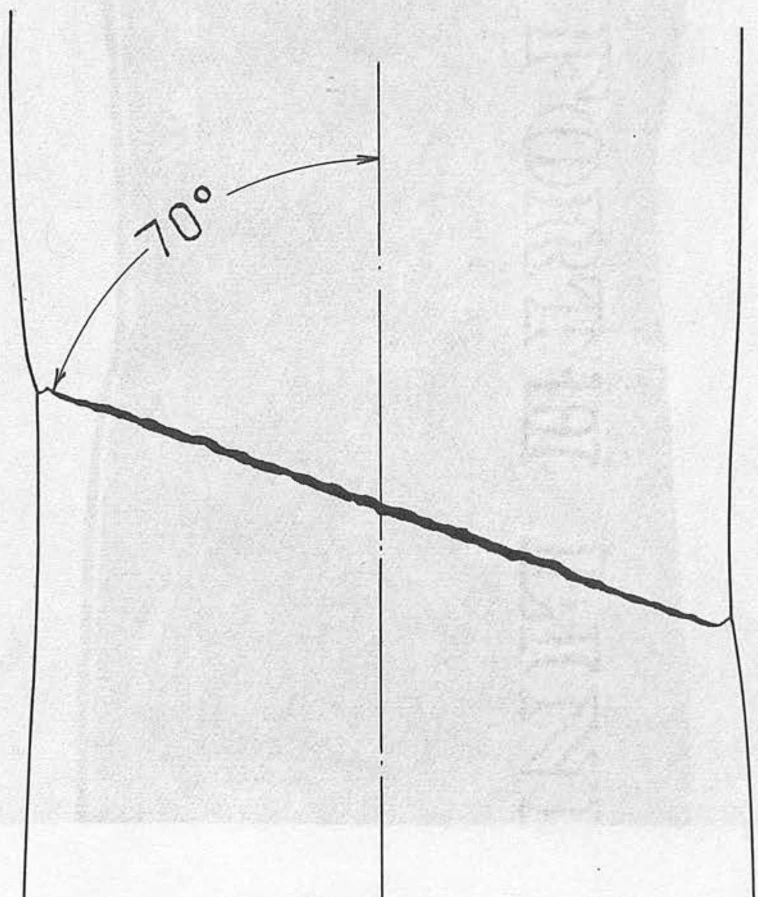

Fig. 34.

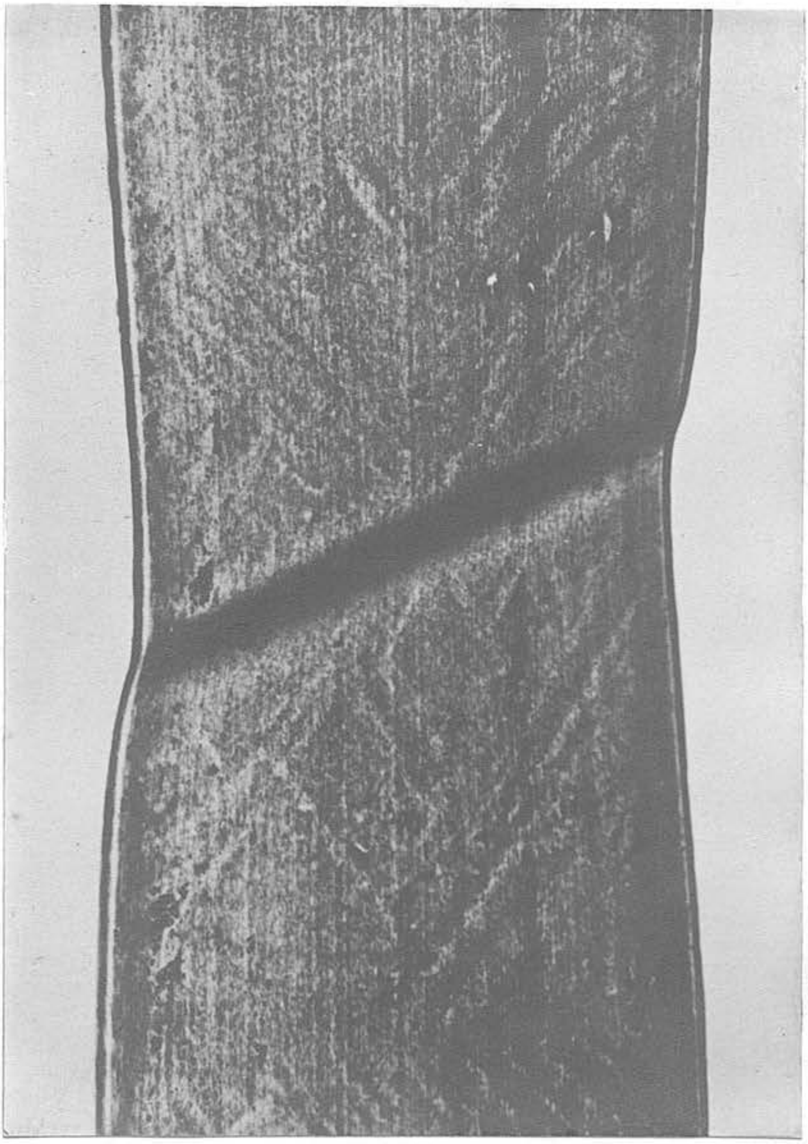

Fig.35

of stress is most marked. The cause of Lüders' lines is well known, but no satisfactory explanation of the contractile phenomenon has yet appeared. In the opinion of the writer the latter is an effect of flow in the contracting zone. It may be mentioned that in the case of copper the traces of the slips do not appear in banded form but as a general roughening of the surfaces, and the full contractile cross is not readily formed.

Fig. 36 is a diagram of one of the $3\frac{1}{2}$-inch by $\frac{1}{4}$-inch set after rupture. Variation in thickness is indicated by contour lines drawn at intervals of 0.02 inch, the contour for 0.21 inch being also given. The line of rupture coincides with the bottom of one groove of the contractile cross. This mode of failure is very frequently, but not invariably, met with in flat specimens having sectional dimensions of such relative magnitude. The position of the other arm of the cross is indicated by the depression along the oblique dotted line. The smaller angle between the depressions and the centre line is again close on 70°. The contour lines were obtained by setting a micrometer-gauge with conical spindle and anvil to the required thickness, locking it, moving it over the previously chalked faces of the fractured portions

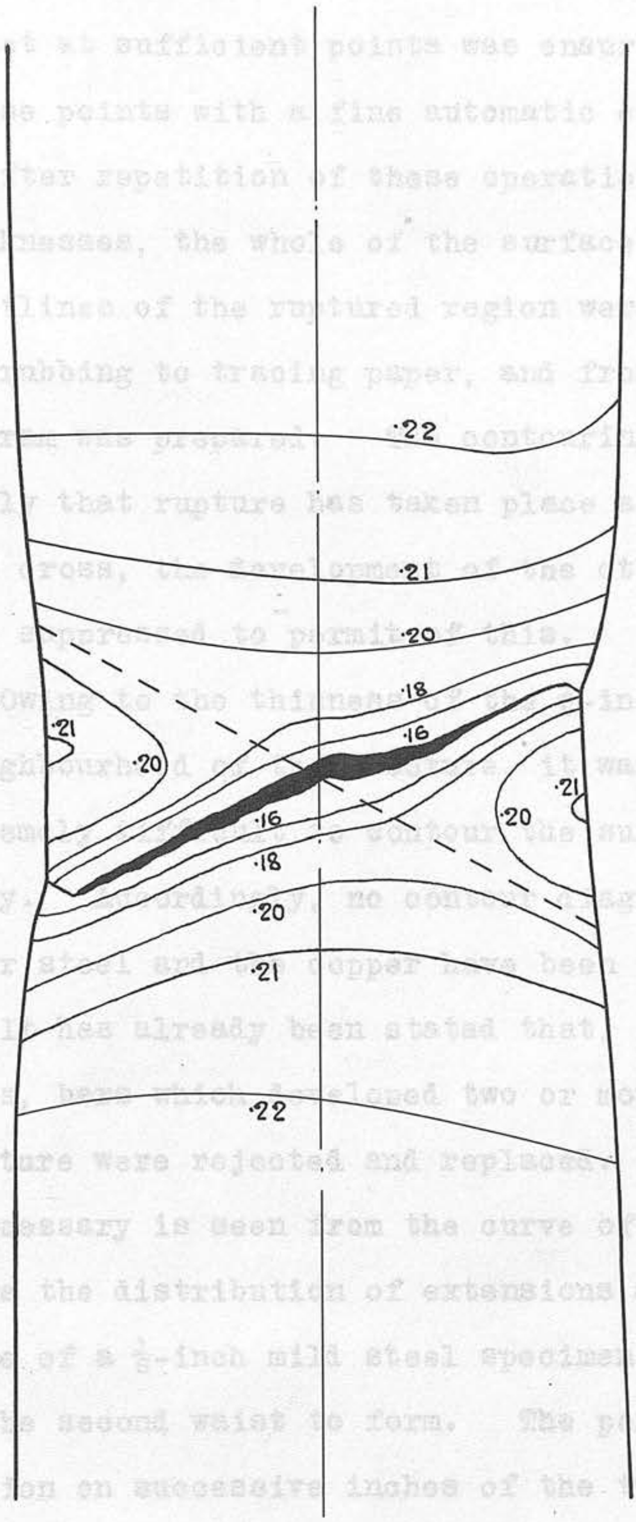

Fig. 36.

till contact at sufficient points was ensured, then fixing these points with a fine automatic centre-punch. After repetition of these operations for other thicknesses, the whole of the surface markings and the outlines of the ruptured region were transferred by rubbing to tracing paper, and from that the final diagram was prepared. The contouring shows very clearly that rupture has taken place along one arm of the cross, the development of the other being apparently suppressed to permit of this.

Owing to the thinness of the $\frac{1}{8}$-inch plates in the neighbourhood of the fracture, it was found to be extremely difficult to contour the surfaces effectively. Accordingly, no contour diagrams for the thinner steel and the copper have been inserted.

It has already been stated that, in these experiments, bars which developed two or more waists before rupture were rejected and replaced. That this is necessary is seen from the curve of Fig. 37 which shows the distribution of extensions along the centre line of a $\frac{1}{8}$-inch mild steel specimen which broke at the second waist to form. The percentages of elongation on successive inches of the twelve clear of the grips are:- 22.5, 25, 29, 31, 29, 20, 17, 20, 27, 66, 47.5, and 24 per cent. These values

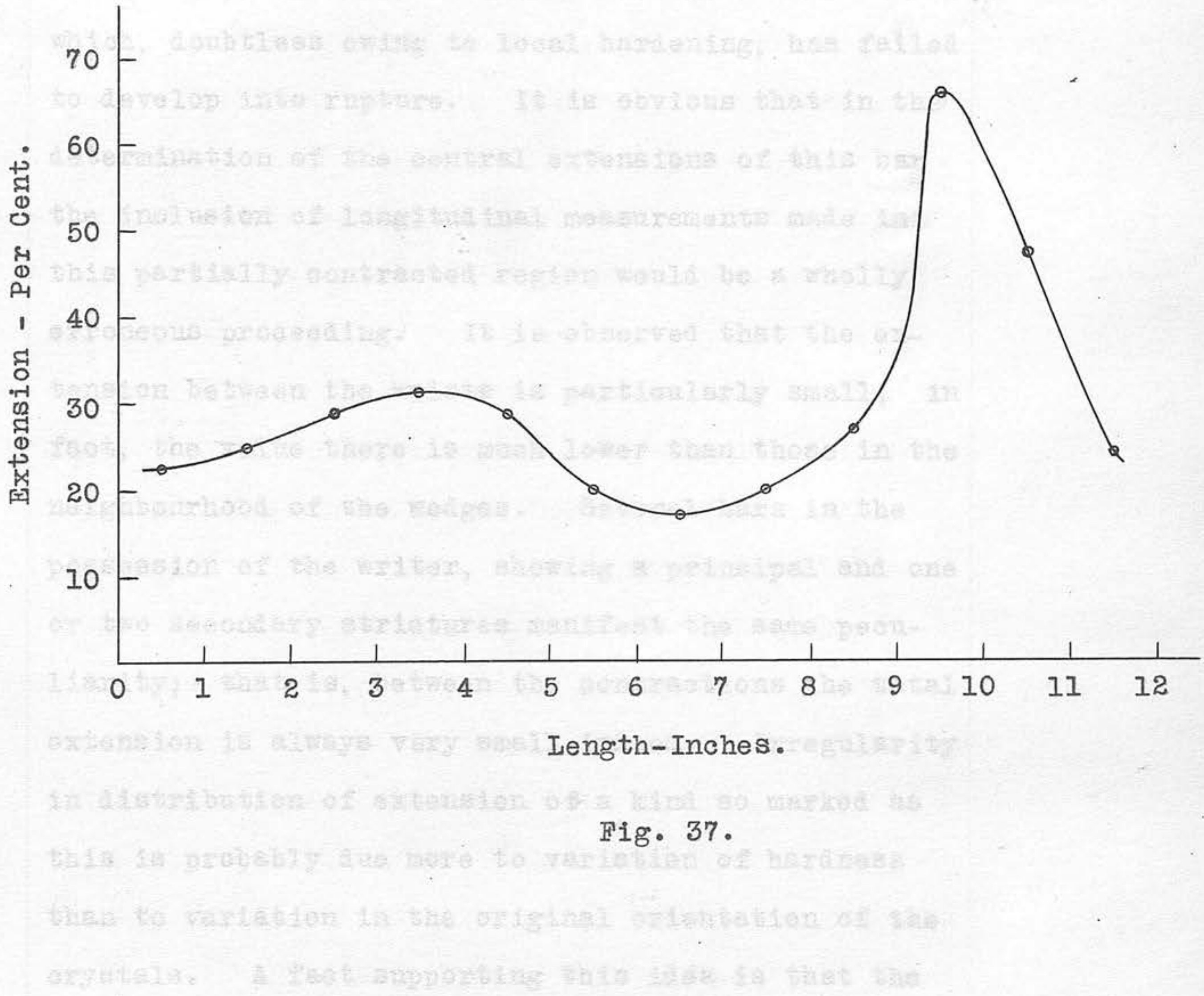
Extension - Per Cent.
70
60
50
40
30
20
10
0
1
2
3
4
5
6
7
8
9
10
11
12
Length-Inches.

Fig. 37.

are plotted over the centre of their respective inches, and the curve of distribution so determined. The higher peak, of course, occurs over the inch within which fracture has taken place: the lower, to the left, indicates a marked secondary stricture which, doubtless owing to local hardening, has failed to develop into rupture. It is obvious that in the determination of the central extensions of this bar the inclusion of longitudinal measurements made in this partially contracted region would be a wholly erroneous proceeding. It is observed that the extension between the waists is particularly small; in fact, the value there is much lower than those in the neighbourhood of the wedges. Several bars in the possession of the writer, showing a principal and one or two secondary strictures manifest the same peculiarity; that is, between the contractions the total extension is always very small indeed. Irregularity in distribution of extension of a kind so marked as this is probably due more to variation of hardness than to variation in the original orientation of the crystals. A fact supporting this idea is that the phenomenon appears to occur most frequently in inferior or non-homogeneous steels.

It is well known that a test-bar of ductile metal ruptures properly in tension not by instantaneous separation but by progression from the axis outwards: rupture is complete in and around the centre while the metal to the outside is still stretching. This results in the familiar phenomenon of the gap in the fracture which is present in all test-pieces of regular section, but which is naturally most visible in the case of flats: its presence in circular and polygonal bars may be immediately verified by cutting a longitudinal axial section. In rectangular specimens tested to destruction, the gap is naturally more marked on the faces than on the edges. Its central breadth, as measured on the longitudinal centre line of the face, depends upon the type of fracture, and is greater in symmetrical than in asymmetrical fractures of the same type. Asymmetrical fractures generally indicate that rupture has not started in the axis, and are doubtless caused by non-axial loading, lack of homogeneity in the material, flaws, etc. In such cases the maximum breadth of the gap is not usually found at the middle of the width, but slightly to one side of it. The presence of a latent defect in the contracting zone may affect to a considerable extent the ultimate shape of the fracture. Although the ideal fracture of a prismatic bar could be obtained only with homogeneous isotropic material subjected to perfectly axial loading, the fractures of well-worked ductile metals, nevertheless, appear to fall into types more or less regular.

Tables X and XIV and Figs. 15, 16, 17, 21, 22, and 23 in Arts. 6 and 7 show the variation of extension with width on gauge-lengths of 8 inches and 11.3 $\sqrt{\text{area}}$ with and without deduction of gap. A trace of parallelism between the curves of Figs. 21, 22, and 23 suggests that the breadth of the gap is approximately proportional to the square root of the cross-sectional area of the bar. As a matter of fact, the writer has found it to be more nearly proportional to the width of the piece. It must be noted, however, that strictly speaking the relation holds only when the gaps are those found in fractures of similar type.

In some testing laboratories it is usual in the case of flat specimens to subtract one-half the breadth of the gap from the actual elongation before computing the extension: the whole gap, as already stated, was subtracted in calculating the extensions recorded above.

Table XXIII gives the average central

TABLE XXIII.

Central Breadth of Gap.

Nominal Width. Inches.	Central Breadth of Gap. Inches.			
	$\frac{1}{4}$-Inch Steel.	$\frac{1}{8}$-Inch Steel.	$\frac{1}{8}$-Inch Copper.	$\frac{1}{2}$-Inch Steel.
$\frac{1}{4}$	-	0.013	0.032	0.002
$\frac{1}{2}$	0.023	0.020	0.033	0.013
1	0.037	0.036	0.045	0.031
$1\frac{1}{2}$	0.066	0.030	0.047	0.045
2	0.076	0.040	0.132	0.068
$2\frac{1}{2}$	0.080	0.046	0.119	0.092
3	0.140	0.045	0.166	0.110
$3\frac{1}{2}$	0.095	0.045	0.124	0.142
4	0.117	0.058	0.200	0.157

breadth of the gap, irrespective of type of fracture, for the several widths of the three materials. In addition, in the last column are listed the corresponding values obtained from specimens of the same widths but cut from a $\frac{1}{2}$-inch plate which was rolled from the same ingot as the $\frac{1}{8}$-inch steel plate of these investigations. The values for these two Dalzell plates have been plotted in Fig. 38 against the nominal width. Attention may be drawn to the fact that although the magnification of the abscissae is two, the vertical measurements are extended to ten times their actual dimension. The linearity of the respective plots is most marked, and suggests that in the case of high-grade metal the central breadth of the gap in the fracture may be taken as approximately proportional to the width of the specimen. The difference in the rate of increase of breadth of gap for the two thicknesses is worthy of note. Apart from the point for the 3-inch width, the values for the $\frac{1}{4}$-inch steel specimens also plot somewhat closely about their best straight line; but, as the plate was of unknown origin and to avoid complication in the diagram, the graph has not been incorporated. The copper once more shows its lack of uniformity, the continuity of the values suffering a break between

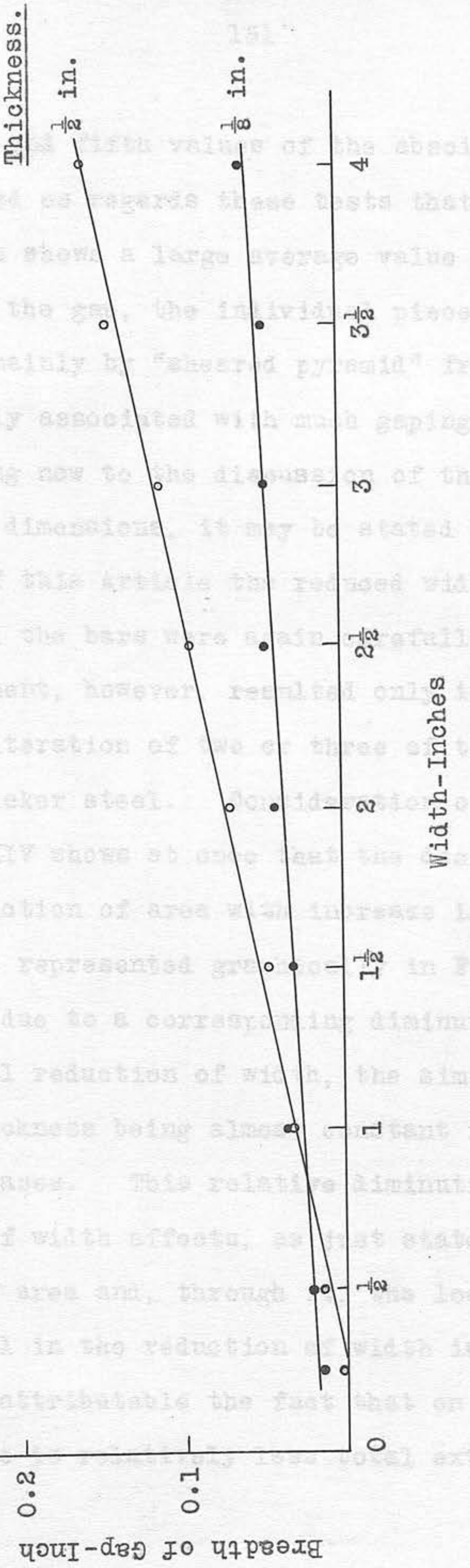
Thickness.
½ in.
⅛ in.
Breadth of Gap-Inch
0.2
0.1
0
½
1
1½
2
2½
3
3½
4
Width-Inches

Fig. 38.

the fourth and fifth values of the abscissa. It may be added as regards these tests that wherever a set of bars shows a large average value for the breadth of the gap, the individual pieces of that set broke mainly by "sheared pyramid" fracture - a type usually associated with much gaping.

Passing now to the discussion of the final transverse dimensions, it may be stated that for the purposes of this Article the reduced width and thickness of all the bars were again carefully determined. Re-measurement, however, resulted only in a very trifling alteration of two or three of the values for the thicker steel. Consideration of the figures of Table XXIV shows at once that the decline in value of the reduction of area with increase in width of test-piece, represented graphically in Figs. 30, 31, and 32, is due to a corresponding diminution in the proportional reduction of width, the similar quantity for the thickness being almost constant in each of the three cases. This relative diminution in the reduction of width affects, as just stated, the contraction of area and, through it, the local extension. To this fall in the reduction of width is therefore ultimately attributable the fact that on a fixed length there is relatively less total extension in a

TABLE XXIV.

Reduction of Width and Thickness.

Nominal Width.	$\frac{1}{4}$-Inch Steel.		$\frac{1}{8}$-Inch Steel.		$\frac{1}{8}$-Inch Copper.	
Inches.	Reduction of Width. Per cent.	Reduction of Thickness. Per cent.	Reduction of Width. Per cent.	Reduction of Thickness. Per cent.	Reduction of Width. Per cent.	Reduction of Thickness. Per cent.
$\frac{1}{4}$	-	-	22.0	51.5	30.0	31.1
$\frac{1}{2}$	31.8	42.7	19.0	51.1	30.9	30.8
1	27.3	41.7	15.9	52.2	28.6	32.3
$1\frac{1}{2}$	23.6	41.8	13.3	52.9	25.7	32.6
2	22.8	41.6	12.7	52.6	24.2	31.6
$2\frac{1}{2}$	21.1	42.5	11.7	52.6	23.2	31.8
3	21.0	41.1	11.0	52.2	23.7	30.6
$3\frac{1}{2}$	19.7	44.4	10.7	51.9	23.6	31.6
4	20.1	42.4	10.9	52.2	22.4	30.6

wide bar than in a narrow one. Further, it is to be noted that the thickness is reduced relatively more than the width and that in general the smaller the initial transverse dimension, the greater its ultimate reduction. These features are very well marked in the case of the steel specimens, but less so in that of the copper bars. The percentage of reduction of width appears to tend with increasing width of piece towards an asymptotic value. On the other hand, the percentage of reduction of thickness is apparently unaffected by the width and, with the exception of the entry for the $3\frac{1}{2}$-inch set of the thicker steel, shows little divergence from the mean value in each case.

As no good purpose would be served by representing graphically the values of the reduction of width - the curves being similar to those for the reduction of area - a slightly different but very effective method has been employed to show the relative diminution of reduction of transverse dimension with increasing width of test-bar. In Fig. 39 the profiles of four of the ruptured specimens of the $\frac{1}{4}$-inch plate are shown drawn to a base of $\frac{\text{length}}{\text{width}}$. The mean outlines of the bars originally $\frac{1}{2}$, 1, 2, and 4 inches wide are drawn to the scale of their

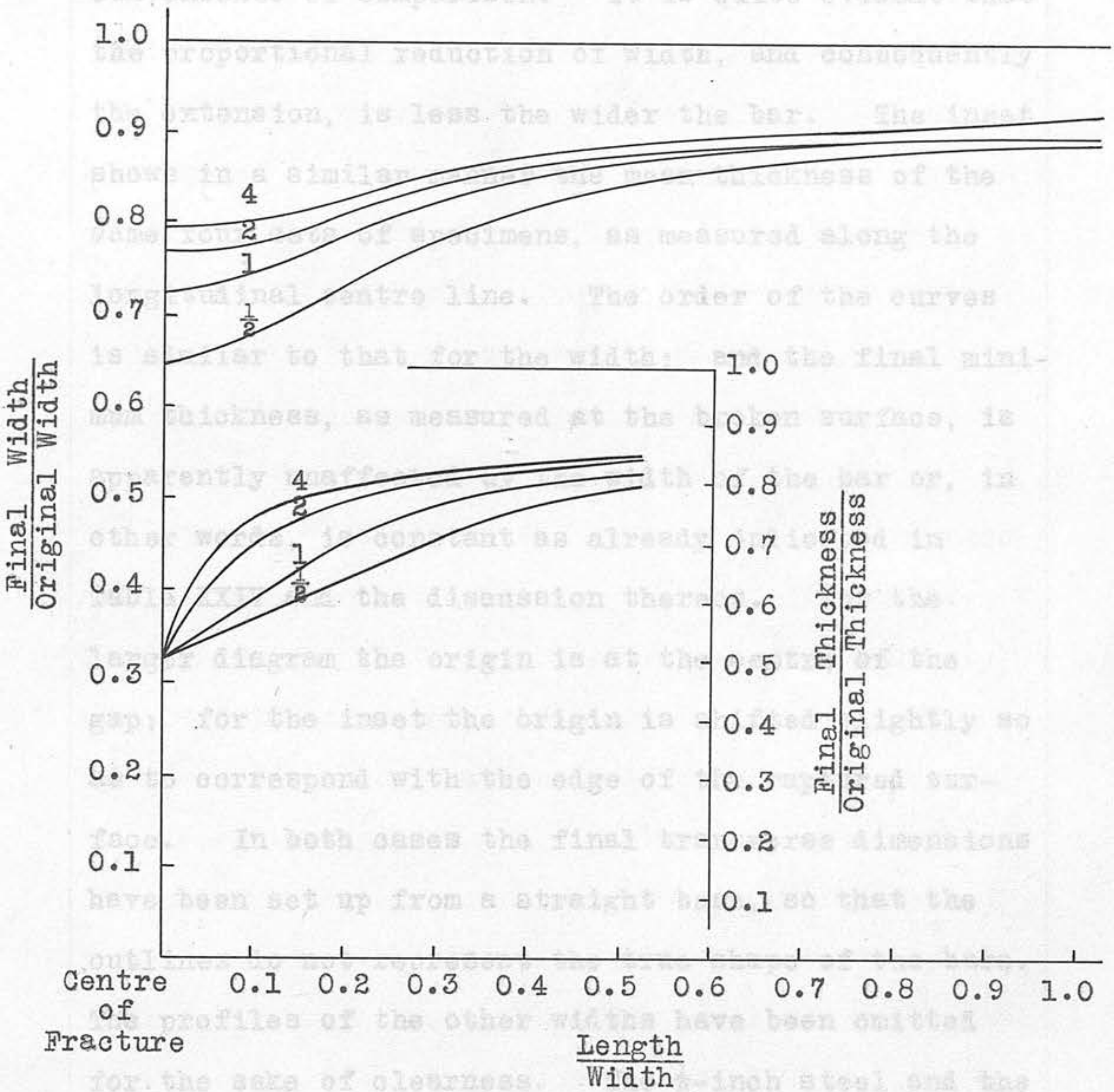
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Final Width / Original Width
4
2
1
½
4
2
1
½
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Final Thickness / Original Thickness
Centre of Fracture
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
Length / Width

Fig. 39.

initial dimensions, all reduced to the same size for convenience of comparison. It is quite evident that the proportional reduction of width, and consequently the extension, is less the wider the bar. The inset shows in a similar manner the mean thickness of the same four sets of specimens, as measured along the longitudinal centre line. The order of the curves is similar to that for the width; and the final minimum thickness, as measured at the broken surface, is apparently unaffected by the width of the bar or, in other words, is constant as already indicated in Table XXIV and the discussion thereon. For the larger diagram the origin is at the centre of the gap: for the inset the origin is shifted slightly so as to correspond with the edge of the ruptured surface. In both cases the final transverse dimensions have been set up from a straight base, so that the outlines do not represent the true shape of the bars. The profiles of the other widths have been omitted for the sake of clearness. The $\frac{1}{8}$-inch steel and the copper specimens gave equally good width-profiles; but, owing to the thinness of the plates, the thickness-profiles were not so successful. In consequence, the corresponding diagrams for these metals have not been incorporated.

The values of the final ratio of width to thickness, both at the fracture and at a point originally distant therefrom a length equal to the width of the bar, and of the same ratio expressed as a percentage of the corresponding initial ratio are given in Tables XXV, XXVI, and XXVII. It is observed that in each case the final ratio at the fracture is greater than the initial ratio - a result proving, as before, that the metal is reduced relatively more in thickness than in width. From Figs. 40, 41, and 42, in which the entries of the last two columns of each of the three Tables are shown plotted against the initial ratio, it is seen that, as regards measurements at the fracture, the change in the ratio increases fairly continuously with the ratio itself. On the other hand, in the sensibly straight portion of the bar, that is, clear of the contracted zone, the change in the ratio is practically constant, the increase in value being about $3\frac{1}{2}$ per cent. throughout in the case of the thicker steel and 10 per cent. in that of the thinner steel: a singular feature in the case of the copper is that repeated measurements failed to show any sensible increase, the graph, accordingly, lying at the 100 per cent. level (Fig. 42). In general, the figures of Table XXVII indicate that

TABLE XXV.

$\frac{1}{4}$-Inch Steel - Final Ratio of Width to Thickness.

Nominal Width. Inches.	Width / Thickness.			Final Ratio / Initial Ratio. Per cent.	
	Initial.	Final.			
		At Fracture.	Outside Stricture.	At Fracture.	Outside Stricture.
$\frac{1}{2}$	1.76	2.10	1.82	119	103
1	3.81	4.75	3.95	125	104
$1\frac{1}{2}$	5.64	7.40	5.85	131	104
2	7.57	10.02	7.86	132	104
$2\frac{1}{2}$	9.57	13.11	9.88	137	103
3	11.14	14.96	11.50	134	103
$3\frac{1}{2}$	13.32	19.25	13.85	144	104
4	15.39	21.36	15.92	139	103

TABLE XXVI.

$\frac{1}{8}$-Inch Steel - Final Ratio of Width to Thickness.

Nominal Width. Inches.	Width / Thickness.			Final Ratio / Initial Ratio. Per cent.	
	Initial.	Final.			
		At Fracture.	Outside Stricture.	At Fracture.	Outside Stricture.
$\frac{1}{4}$	1.81	2.91	2.02	161	111
$\frac{1}{2}$	3.66	6.06	4.02	166	110
1	7.35	12.94	8.14	176	111
$1\frac{1}{2}$	10.98	20.25	12.06	184	110
2	14.65	26.95	16.16	184	110
$2\frac{1}{2}$	18.25	33.95	20.00	186	110
3	21.99	40.95	23.97	186	109
$3\frac{1}{2}$	25.96	48.14	28.18	186	109
4	29.89	55.77	32.87	187	110

TABLE XXVII.

$\frac{1}{8}$-Inch Copper - Final Ratio of Width to Thickness.

Nominal Width. Inches.	Width / Thickness.			Final Ratio / Initial Ratio. Per cent.	
	Initial.	Final. At Fracture.	Final. Outside Stricture.	At Fracture.	Outside Stricture.
$\frac{1}{4}$	1.90	1.93	1.90	101	100
$\frac{1}{2}$	3.76	3.76	3.74	100	99
1	7.69	8.11	7.79	105	101
$1\frac{1}{2}$	11.26	12.40	11.28	110	100
2	15.04	16.65	15.01	111	100
$2\frac{1}{2}$	18.49	20.87	18.31	113	99
3	22.34	24.52	22.13	110	99
$3\frac{1}{2}$	26.22	29.24	26.41	112	101
4	29.83	33.34	29.75	112	100

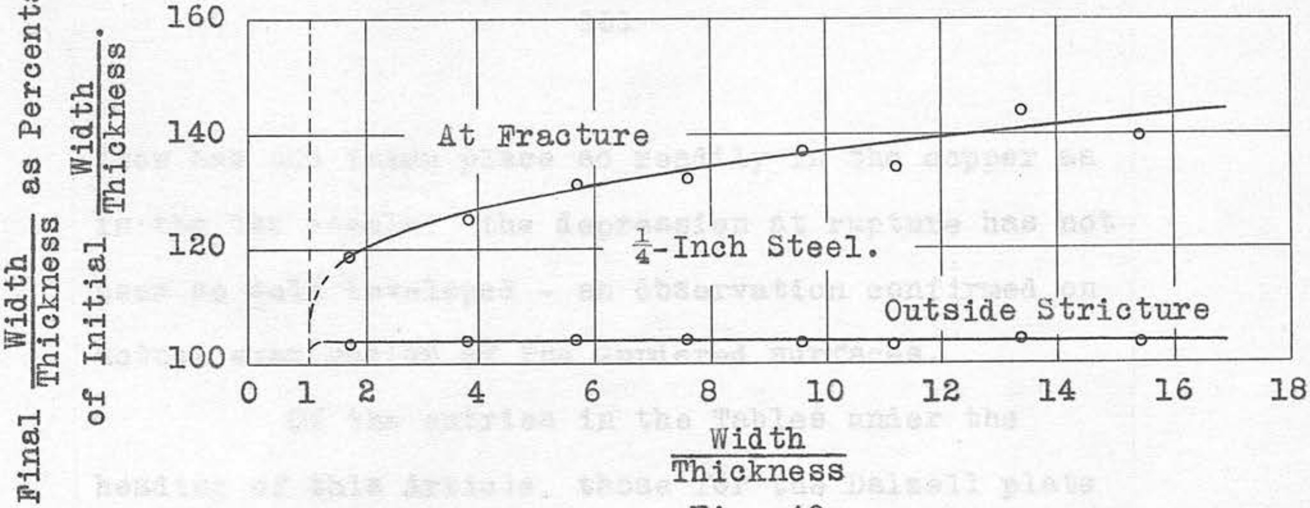

Fig. 40.

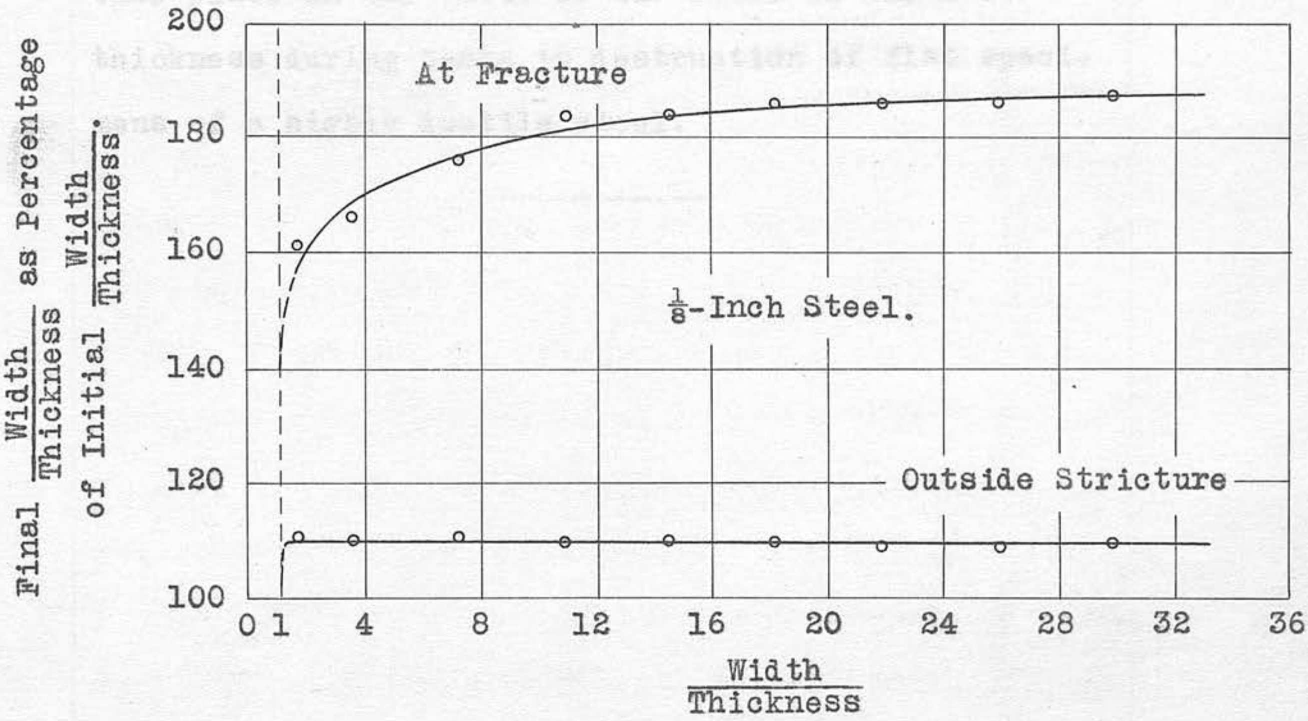

Fig.41.

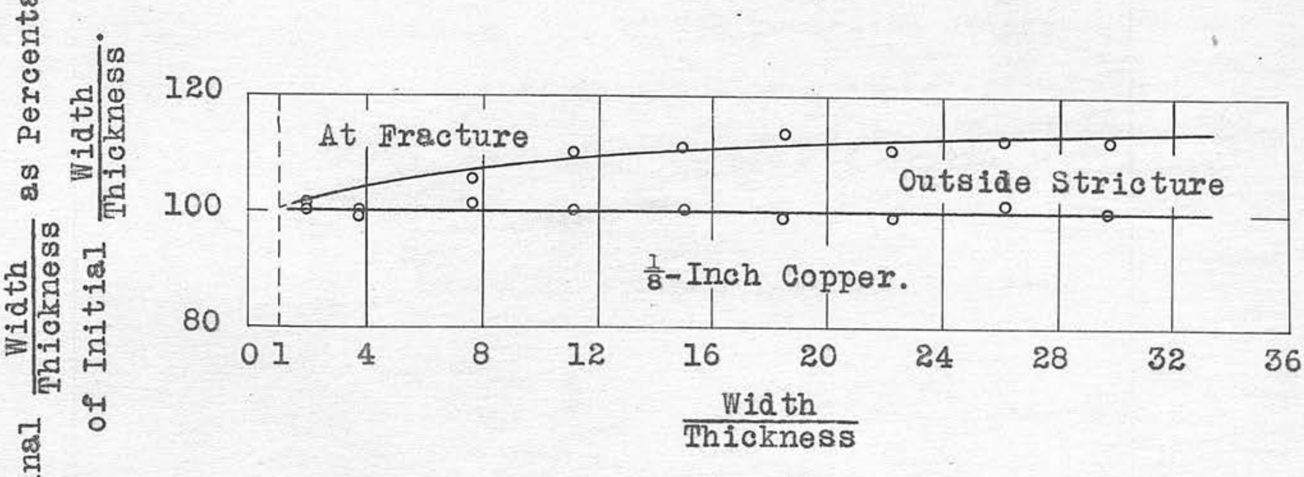

Fig.42.

flow has not taken place so readily in the copper as in the two steels: the depression at rupture has not been so well developed - an observation confirmed on actual examination of the sundered surfaces.

Of the entries in the Tables under the heading of this Article, those for the Dalzell plate may be taken as representative of the changes that take place in the value of the ratio of width to thickness during tests to destruction of flat specimens of a highly ductile steel.

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12. Summary.

The research originated in an attempt to re-establish the apparently critical ratio of width to thickness which Barba found to give, in the case of flat test-bars of mild steel, a maximum of extension on fixed or constant gauge-lengths, but the scope of the investigation ultimately became much wider.

The main object of the experiments discussed in this Thesis was to determine the nature of the influence of width of specimen upon the results of tensile tests of mild steel and rolled copper. The test-bars were cut from $\frac{1}{4}$-inch and $\frac{1}{8}$-inch steel boiler plates and from $\frac{1}{8}$-inch copper plate, and were machined to give widths varying from $\frac{1}{4}$ inch to 4 inches, the range of the ratio $\frac{\text{width}}{\text{thickness}}$ in the thinner specimens being approximately from 2:1 to 30:1.

The results indicate the influence of width of specimen on the strength, on the extension of fixed lengths, on the extension of three types of variable gauge-length, and on the reduction of area and certain other quantities involving the final transverse dimensions. For the most part, the equations were derived from the results obtained from the specimens of the high-grade $\frac{1}{8}$-inch steel plate.

Further research on similar lines carried out on $\frac{1}{4}$-inch, $\frac{3}{8}$-inch, and $\frac{1}{2}$-inch mild steel plates, all of which were rolled from slabs cut from the same ingot, gave evidence definitely corroborative of the views latterly expressed by the writer; but to avoid unnecessary repetition no record of that investigation has been included in the present exercise.

The leading results are summarised under the headings of the Articles in which they appear.

Yield-Point, Tenacity, and Mean Breaking Stress.

(1) The yield-point remains practically constant over the entire range of observation in each case, the average values being 18.14 tons per square inch for the $\frac{1}{4}$-inch steel, 12.82 for the $\frac{1}{8}$-inch steel, and 7.32 for the $\frac{1}{8}$-inch copper.

(2) The tenacity also appears to be but little affected, the average values being 25.48 tons per square inch for the thicker steel, 22.37 for the thinner steel, and 14.52 for the copper.
Width of specimen would therefore appear to have no appreciable influence on the strength, elastic and ultimate, of mild steel or rolled copper.

(3) The values of the mean ultimate breaking stress for the $\frac{1}{4}$-inch steel vary considerably, but those for the $\frac{1}{8}$-inch steel and the copper show some approach to constancy. The average values for the three series of tests are 49.8, 46.4, and 25.8 tons per square inch respectively.

Extension on Fixed Lengths.

(1) A modification of the ordinarily accepted extension-length relation is given. The

The results suggest that the constant b in the usual equation conceivably contains more than the true general extension.

(2) The extension on 8 inches is considerably influenced by width of specimen, rising at first somewhat quickly but subsequently more slowly. The maximum variation of value in the case of the thicker steel amounts to 10 per cent. or nearly one-half of the lowest value of extension in the series; in that of the thinner steel to 7 per cent. or almost one-third of the lowest value; and in that of the copper to 13 per cent. or roughly two-fifths of the lowest value.
Simple parabolic, logarithmic, and exponential equations have been successfully fitted to the results for this important gauge-length.
On the assumption that the wavelike form of the curves is due to the presence of a periodic element, a tentative theory is advanced which, however, the later investigation, referred to above, failed to substantiate so far, at least, as the quasi-harmonic aspect is concerned.

(3) The extension on other fixed lengths varies in a closely similar manner.

(4) The extension on zero length or the limiting extension in the fracture falls with increasing width of specimen, the relation being well represented by an exponential or a simple logarithmic equation.

Extension on Lengths Proportional to $\sqrt{\text{Area}}$.

(1) The extension on the German standard gauge-length of 11.3 $\sqrt{\text{area}}$ is much less influenced by width of specimen than that on 8 inches, falling only very slightly as the width is increased. The difference between the extreme values is considerably reduced, the maximum variation in the case of the thicker steel amounting to only 2 per cent. or one-fourteenth of the lowest value of extension in the series; in that of the thinner steel

to 3 per cent. or roughly one-tenth of the lowest value; and in that of the copper to 7 per cent. or about two-elevenths of the lowest value.
The results for this particular gauge-length have been successfully treated on two straight-line bases - one horizontal, the other slightly oblique.

(2) The extension on other lengths proportional to $\sqrt{\text{area}}$ varies in a closely similar manner.

(3) For reasonably large values of the coefficient of $\sqrt{\text{area}}$, the influence of width of specimen is so small as to be commercially negligible.

Extension on Lengths Proportional to the Width.

(1) The extension on a gauge-length equal to some definite proportion of the width, or of the ratio $\frac{\text{width}}{\text{thickness}}$, expressed in inches, decreases as the width, or the ratio itself, increases.

(2) The types of equation best representing the relation in this instance are the exponential and the logarithmic.

(3) The magnitude of the variation of extension is such as to indicate that comparable results are not to be expected from this type of gauge-length.

Extension on Lengths Proportional to Area/Perimeter.

(1) The extension on gauge-lengths (in inches) proportional to the ratio $\frac{\text{area}}{\text{perimeter}}$ rises with increasing width of bar in a manner somewhat similar to that for fixed lengths.

(2) Simple parabolic and logarithmic curves have been successfully fitted to the observed values.

(3) The results do not indicate a degree of comparability sufficiently high to merit the suggestion of the use of this class of gauge-length in practice.

Reduction of Area.

(1) The percentage of contraction of area falls at first quickly and subsequently more slowly with increase in width of specimen. The extreme difference in value is 10 per cent. in the case of the $\frac{1}{4}$-inch steel, 5 per cent. in that of the $\frac{1}{8}$-inch steel, and 6 per cent. in that of the copper. These percentages correspond to two-elevenths, one-eleventh, and two-fifteenths of the lowest value of reduction in their respective series.

(2) The reduction-width relation is particularly well represented by simple equations of exponential and logarithmic type.

(3) The sum of the extension on one of the larger fixed lengths and the corresponding reduction of area is practically constant, the amount of the maximum deviation from the mean over the entire range, being in general around 1 per cent. The sum of the two most important commercial criteria of ductility is thus almost independent of the width of the piece.

The Final Ratio of Width to Thickness.

(1) With increase in width of specimen, the line of rupture on the face passes from the symmetric, through the quasi-symmetric, to the wholly asymmetric type in which the bar breaks along one contractile groove at about 70° to the direction of stress.
Contouring of the widest bars appears to indicate that rupture along one arm takes place only by the suppression of the development of the other arm of the contractile cross.
Lüders' lines and the contractile depressions are distinct phenomena.

(2) Secondary strictures are probably due more to variation of hardness than to variation in the original orientation of the crystals.

(3) The central breadth of the gap in the fracture is very nearly proportional to the width of the specimen.

(4) The percentage of reduction of width falls with increasing width of test-piece; but the similar quantity for the thickness remains practically constant, and its value is greater. Hence, the thickness of a flat specimen is reduced relatively more than the width, and the width of a narrow bar is reduced relatively more than that of a wide one; or, in general, the smaller the initial transverse dimension the greater its ultimate reduction.
The relative diminution in the reduction of width affects the contraction of area and, through it, the local extension. To this fall in the reduction of width is therefore ultimately attributable the fact that on a fixed length there is relatively less total extension in a wide bar than in a narrow one.

(5) The value of the final ratio of width to thickness at the fracture, expressed as a percentage of the initial ratio, rises with the width; but in the portions of the bar outside the stricture the change in the ratio is constant.

Returning to the starting point of the research, it may with reason be stated that Barba's critical ratio is merely the very variable upper limit of the period of continuous increase of extension: beyond this point irregularity in value is manifested. The causes of its existence, if present, are mechanical rather than geometrical.

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